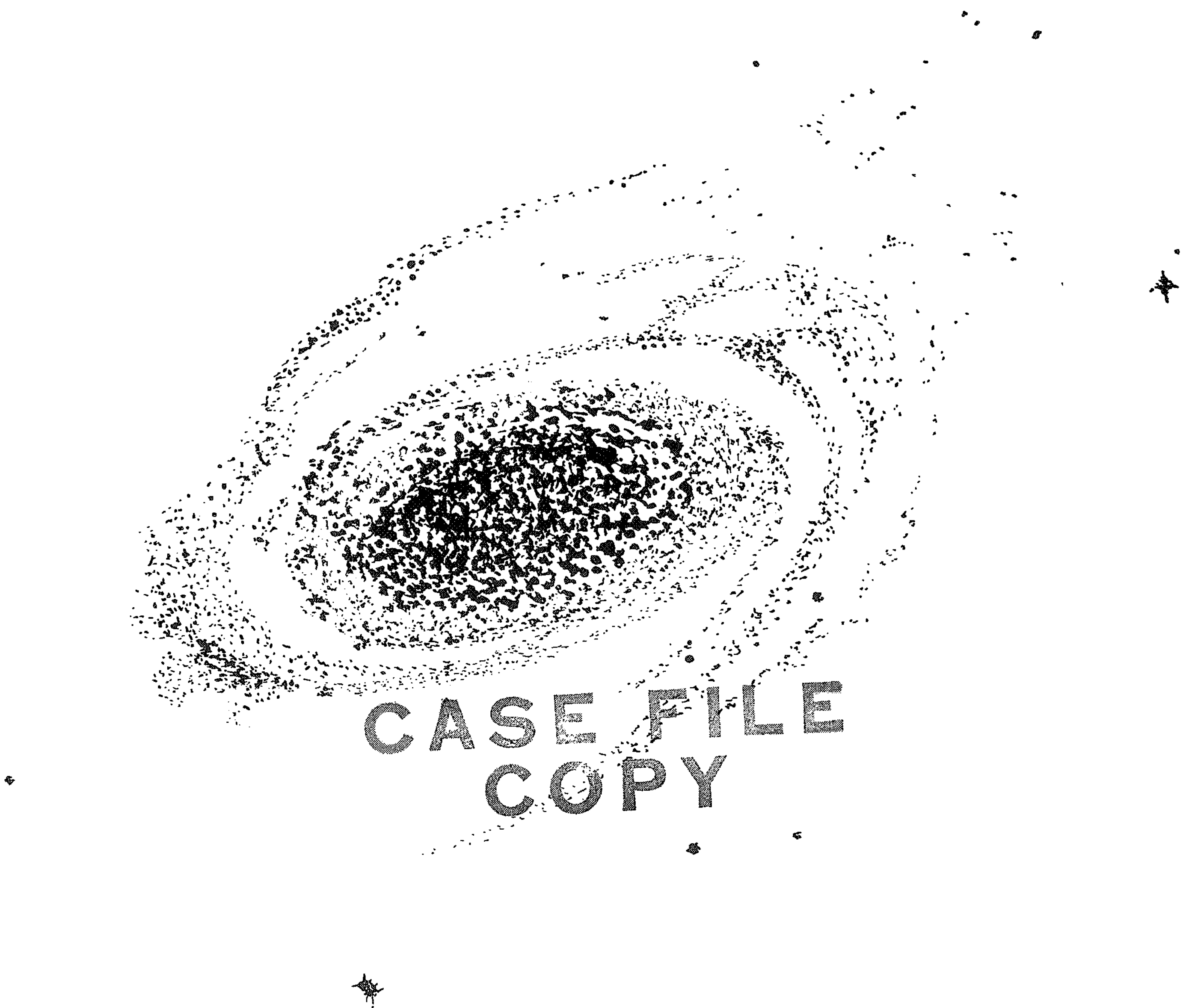
N70-32815

FIREBALLS AND THE PHYSICAL THEORY OF METEORS

R. E. McCROSKY and Z. CEPLECHA



Smithsonian Astrophysical Observatory
SPECIAL REPORT 305

Research in Space Science
SAO Special Report No. 305

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Richard E. McCrosky and Zdeněk Ceplecha

November 19, 1969

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

910-153

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ABSTRACT

We recognize a number of uncertainties and inconsistencies in the classical theory of meteors. However, the general characteristics of the theory are verified by a sizable class of objects of moderate brightness and we therefore accept it as a first approximation.

We believe the bulk density of the meteoroid to be the least well-determined parameter entering the theory and one of the most important. If we could demonstrate convincingly that densities for some class of objects are similar to the density of meteoritic stone, we could accept the composition and structure as (probably) known and reduce the uncertainties in the physical theory of meteors. On the other hand, low-density material is predicted by comet models, and proof of its existence is of substantial importance. We thus think it unjustified to assume a high density for meteoritic material. A straightforward interpretation of meteor data has, in the past, suggested that low density prevailed. The same result is now seen in the fireball data.

We have attempted to alter the classical theory, following other authors in part, in order to explain observations of faint and bright meteors in terms of a high density. None of the explanations offered by previous authors can be successfully extrapolated to the very bright fireballs. Frothing of the meteoroid as suggested by Allen and Baldwin becomes less important as the body size increases and less possible as the object penetrates deeper into the atmosphere. Fragmentation by thermal shock as proposed by Jones and Kaiser becomes decreasingly important as the body size increases, as can be demonstrated both by a mathematical model and by the existence of meteorites of less than the critical size.

We have also treated three additional variations in the theory. These are fragmentation of small particles, gross fragmentation, and a reverse-rocket effect produced by high-velocity spall. We find all of these to be either inefficient or unrealistic models for disguising the true bulk density.

We conclude either that almost all objects are low density or that the meteor theory or the constants employed contain a gross error. We consider this latter possibility to be slight.

The small terminal masses of most fireballs lend support to our contention that they are unlike meteorites. The near absence of any large masses over a 5-year period in the Prairie Network casts serious doubt on our predicted rate of fall of meteorites. We have noted that three quite different physical effects — thermal shock, ablation, and pressure fragmentation — may produce substantial variations between the mass-number flux of meteorites outside the atmosphere and on the ground, and we believe that it is impossible at present to make a sensible extrapolation from the observed distribution to that in space.

RESUMÉ

Nous identifions un certain nombre d'incertitudes et d'inconsistances dans la théorie classique des météores. Cependant, les caractéristiques générales de la théorie étant vérifiées par une classe assez importante d'objets de brillance modérée, nous l'acceptons donc comme une première approximation.

Nous pensons que la densité de masse de la météorite est le paramètre, relatif à la théorie, le moins bien déterminé et un des plus importants. Si nous pouvions démontrer d'une façon convaincante que les densités pour certaines classes d'objets sont semblables à la densité de la pierre météoritique, nous pourrions admettre la composition et la structure comme connues (probablement) et réduire les incertitudes dans la théorie physique des météores. D'un autre côté, les modèles de comètes prédisent une matière à faible densité, et la preuve de son existence est d'une considérable importance. Nous pensons donc qu'il n'est pas justifié de présumer que la matière météoritique ait une forte densité. Une interprétation directe des données des météores, dans le passé, a suggéré qu'une faible densité prévalait. Le même résultat est observé maintenant dans les données des globes de feu.

Adoptant partiellement les idées d'autres auteurs, nous avons essayé de changer la théorie classique pour expliquer les observations de météores faibles et de météores brillants en tant qu'objets à forte densité. Aucune des explications offertes par les auteurs précédents ne peut être extrapolée avec succès aux globes de feu très brillants. La formation d'écume sur la météorite, suggérée par Allen et Baldwin, devient moins importante quand la taille de l'objet augmente et moins probable quand l'objet pénètre plus profondément dans l'atmosphère. La fragmentation par choc thermique, comme celle proposée par Jones et Kaiser, décroit en importance quand la taille de l'objet augmente, comme on peut le démontrer par un modèle mathématique et aussi par l'existence de météorites plus petites que la taille critique.

Nous avons aussi traité trois variations supplémentaires dans la théorie. Ce sont la fragmentation de petites particules, la fragmentation grossière, et un effet de rétrofusée produit par un éclat de grande vitesse. Nous trouvons que toutes ces variations sont des modèles soit inefficaces soit irréalistiques pour dissimuler la vraie densité de masse.

Nous concluons que presque tous les objets sont de faible densité ou bien que la théorie des météores, ou les constantes employées, contiennent une erreur flagrante. Nous pensons que cette dernière possibilité est très petite.

Le fait que les masses finales de la plupart des globes de feu soient petites, a apporté un support à notre affirmation que ces globes sont différents des météorites. L'absence presque totale de grandes masses pendant 5 ans dans le Prairie Network fait planer un doute sérieux sur le flux des tombées de météorites que nous avons prédit. Nous avons noté que trois effets physiques tout à fait différents - choc thermique, ablation et fragmentation par pression - peuvent produire d'importantes variations entre les flux du nombre et de la masse des météorites en dehors de l'atmosphère et sur le sol, et nous pensons qu'il est impossible actuellement d'extrapoler raisonnablement la distribution observée pour obtenir celle dans l'espace.

КОНСПЕКТ

Мы осознаем число неопределенностей и непоследовательностей, которые содержатся в классической теории метеоров. Однако, общие характеристики теории подтверждаются значительным классом объектов умеренной яркости и поэтому мы принимаем ее за первое приближение.

Мы считаем, что основная плотность метеорного тела является наименее определенным параметром в теории и одним из наиболее важных. Если бы мы смогли убедительно продемонстрировать сходство между плотностью некоторого класса объектов и плотностью метеоритного камня, мы смогли бы принять состав и структуру за (вероятно) известные и уменьшить неопределенности в физической теории метеоров. С другой стороны, материал с низкой плотностью предсказывается кометными моделями, а доказательство его существования является вопросом исключительной важности. Таким образом, мы считаем неоправданным допускать высокую плотность у метеоритных материалов. Прямая интерпретация метеорных данных в прошлом указывала на преобладание низкой плотности. Те же результаты наблюдаются и в данных болидов.

Мы пытались изменить классическую теорию, частично следуя другим авторам, чтобы объяснить наблюдения слабых и ярких метеоров с точки зрения высокой плотности. Ни одно из объяснений, предложенных предыдущими авторами, не может быть экстраполировано к случаю очень ярких болидов. Вспенивание метеоров, как предложено Алленом и Болдуином, становится менее важным с увеличением размера тела и менее возможным с более глубоким проникновением объекта в атмосферу. Фрагментация тепловым ударом, как предложено Джонсоном и Кайзером, становится менее важной с увеличением размера тела, что может быть продемонстрировано как математическими моделями так и существованием метеоритов размером меньше критического.

Мы также рассматривали три дополнительные вариации в теории, такие как: фрагментация малых частиц, массовая фрагментация и реактивный эффект, производимый осколком, двигающимся с высокой скоростью. Мы находим все эти модели или недостаточными или нереальными для объяснения действительной массовой плотности.

Мы делаем вывод: или почти все объекты имеют низкую плотность или электронная теория и входящие в нее постоянные глубоко ошибочны. Мы считаем последнюю возможность мало вероятной.

Небольшие терминальные массы большинства болидов поддерживают наше положение о том, что они не похожи на метеориты. Почти полное отсутствие любых больших масс за 5-летний период работы сети Прэри оставляет серьезные сомнения по поводу предсказанного хода выпадений метеоритов. Мы заметили, что три довольно различные физические эффекты-тепловой удар, абляция и фрагментация за счет давления-могут привести к значительным изменениям в масс-числе потока метеоритов вне атмосферы и на земле; мы уверены, что в настоящее время невозможно провести разумную экстраполяцию от обозреваемого распределения к распределению в атмосфере.

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1. INTRODUCTION

Information available from meteor photographs, together with present knowledge of the physics of meteor phenomena, is insufficient to determine uniquely the physical parameters that describe a meteoroid. The range of solutions consistent with the data heretofore available extends from low-density meteoroids whose structure has minimal integrity (Jacchia, 1955; Jacchia et al., 1967) to meteoroids whose structure may be similar to that of some chondritic meteorites (Allen and Baldwin, 1967; Jones and Kaiser, 1966). The success of all these models rests on the introduction of some new variables in the classical equations. These variables — progressive fragmentation, frothing, or thermal shock — are not in themselves limited by any known parameters of the meteoroid and thus provide sufficient leeway to explain almost any of the observed phenomena among fainter meteors. The basic difference in approach of the above authors can be expressed in terms of the bulk density of the meteoroid material. Jacchia et al. (1967) consider this to be a free parameter to be determined by the observations, while the others accept, a priori, a density and structure of meteoritic stone. A resolution of this problem thus contains an answer to the question of whether or not there exists in the solar system material of grossly different structure than that represented in our meteorite collections. A cometary source for many meteors is indisputable. The generally accepted model of a comet (Whipple, 1951) predicts low-density meteoroids. A cometary origin has also been suggested for meteorites (Öpik, 1966), although perhaps the asteroids remain a more popular source.

This work was supported in part by grant NGR 09-015-004 from the National Aeronautics and Space Administration.

The purpose of this paper is to investigate the new observational material of very bright meteors provided by the Prairie Network and to derive additional constraints on the meteor theory or on the structure of the meteoroid. Although we cannot resolve the question of the origin of meteorites with the new information, we can increase our confidence in the low-density interpretation of a significant part of the faint-meteor data.

In Section 2 we review the classical single-body theory and point out its major uncertainties and inconsistencies. In Section 3 we investigate variations on the classical theory when a meteoroid ablates by fragmentation. We will consider six models of fragmentation, including reviews of three models proposed by others to explain the anomalous deceleration observed in faint meteors. Our primary concern here is to demonstrate that all previous suggestions are applicable to small bodies only and that they cannot be important for the much larger objects now under observation by the Prairie Network. We will also outline, and discard, a new suggestion to explain the apparent low densities of meteors of any size.

In Section 4 certain Harvard-Smithsonian data on photometric and dynamic masses of meteoroids are presented and compared. The various interpretations of these results are summarized.

An independent analysis, in Section 5, of the terminal (dynamic) masses of Prairie Network fireballs is used to strengthen the suggestion that these objects have little structural integrity. We conclude with a discussion of the disruptive forces in the atmosphere that influence the number distribution of meteorite masses.

2. THE SINGLE-BODY THEORY

The deceleration and mass-loss equations governing the meteor's trajectory are conventionally written in the form (see list of symbols)

$$\text{(1)} \qquad \dot{V} = -\frac{\Gamma A \rho V^2}{m_d} \quad ,$$

$$\text{(2)} \qquad \dot{m}_d = -\frac{\Lambda A \rho V^3}{2\zeta}$$

Equation (2) is derived from the conceptually pleasing but unproved assumption that mass loss is proportional to the energy flux to the body. We will defend the adequacy of the relation later. In any case, equation (2) is valid only after a preheating period when the temperature of the surface is raised to a value that permits ablation. At any time, a correct expression for the energy flux available for the heating or for the ablation is given by

$$\text{(3)} \qquad \frac{dE}{dt} = \frac{\Lambda A \rho V^3}{2} - \iint \sigma_R \epsilon \left\{ \left[T(S) \right]^4 - T_0^4 \right\} dS \quad ,$$

where the integral is to be taken over the surface S, and σ_R is the Stefan-Boltzmann constant. The neglect of the radiation term in the usual mass-loss equation properly reflects its unimportance for any conceivable material ($T < 3000\,^\circ K$) when the term is compared with the probable energy flux ($\Lambda > 0.01$) on the meteoroid during most of its luminous trajectory.

A complete expression for the deceleration equation can be written as

$$(4) \qquad m_d \dot{V} + \Gamma A \rho V^2 - m_d g \cos Z_R + f \dot{m}_d w = 0 \quad .$$

The gravity term is negligible except during the earliest portion of the trajectory. It will be ignored throughout this paper. The last term is a formal expression for the momentum transferred to the body by the ablated material, where w is the velocity of the ablation products with respect to the meteor, and $-1 \leq f \leq 1$ is a parameter describing the anisotropy of the ablation. The extreme values of f are in force when all material leaves with the velocity vector opposite to or in the same direction as the meteor velocity vector. For isotropic ablation, $f = 0$. The positive values of f (the "reverse-rocket" effect) have been found to be small or, at most, comparable with the first term in the drag equation (Levin, 1961).

With sufficient knowledge of the meteoroid and the meteoric process, either equation (1) or (2), together with the usual observations, could be used to determine the mass of the meteoroid. Since we do not have this knowledge, some additional observational data are required. Meteor spectra demonstrate that essentially all the luminosity is produced by meteoric atoms, and this suggests that the meteor intensity I is proportional to the rate of mass loss by vaporization:

$$(5) \qquad I \propto \dot{m}_v \quad .$$

The vaporization mass loss $\dot{m}_v$ is not necessarily equivalent to the mass-loss term in equation (2), which may include loss of molten or solid material as well as vapor. The single-body theory assumes that $\dot{m}_v = \dot{m}_d$, and we will impose that condition now and in Section 3.4 treat one case where $\dot{m}_v \neq \dot{m}_d$. The relationship (5), based on observations of relatively bright meteors $(-10 < M < -2)$, has evolved to an explicit luminosity-mass law of the form

(6) $$I = \frac{\tau_0}{2} \dot{m}_v V^n \quad .$$

If the observations give intensity and velocity as a function of time, a photometric mass (indicated by the subscript p) can be determined from

(7) $$m_p (t) = \frac{2}{\tau_0} \int_{t_{end}}^{t} \frac{I(t)}{[V(t)]^n} dt \quad .$$

The limit t_{end} is generally taken to be the end of the luminous trajectory. In writing equation (7) we have tacitly assumed that the entire meteoroid mass is vaporized. Since some terminal mass (unvaporized remnants) is to be expected, the photometric mass is a lower limit to the true mass. Though commonly used, this luminosity equation is at best a rough approximation. The form of the equation is simple not because it represents a simple process but rather because the available observations made of an extremely complex process are sufficient to define it in only general terms. Certainly the body size, the air density, and the meteoroid composition play a role in determining the luminosity, and it is inconceivable that the effect of velocity can really be described by a simple exponent. But since the present unknowns, n and the luminous efficiency τ_0, cannot yet be specified with great precision, a more complex formulation can be justified only if new types of observations or more information on the physical process becomes available.

The preceding defense of the simple approach to the luminosity problem requires a rebuttal. The initial statement, relating the intensity to the meteor mass, is known to be incomplete in three respects:

(A) The spectral observations that form the observational basis for equation (5) are of very much brighter meteors than many of the photographic objects analyzed by means of equation (6). Although it may be true that these fainter meteors behave in the same fashion, there is as yet no observational evidence to verify this.

(B) Meteor spectra at times show gross changes in the level of radiative excitation over the meteor trajectory. It is unlikely that the supposed constant τ_0 does not also vary along the trajectory. An additional term, possibly dependent on the air density, might be justified.

(C) Meteor flares represent an abrupt quantitative change in the total meteor luminosity. They often also display a qualitative change in the radiating species. The luminous efficiency of a flare where Ca II is predominant is probably appreciably higher than in the region preceding the flare. Perhaps then the intensity is sometimes dependent on more than the first power of $\dot{m}_v$ in equation (6).

In the following pages, we make the specific assumptions that the preceding inconsistencies and doubts can be ignored in any "first-order" theory and that the photometric mass, as defined, is a good representation of the actual meteoroid mass. In justifying this approach we note that there are many meteors of intermediate brightness (small-camera meteors, $M \approx -5$) for which the observed intensity and velocity over the entire trajectory can be well represented by the classical theory if it is assumed that $m_p = m_d$. The fit is obtained by making the appropriate choice of only two parameters* that remain constant over the trajectory and that maintain reasonably similar values from meteor to meteor. These parameters, σ and K, are derived from equations (1), (2), and (7) and a definition of the frontal area as

$$(8) \qquad \mathcal{A} = A m^{2/3} \rho_m^{-2/3} \quad ,$$

where A is the shape factor. Expressed in terms of the observable quantities,

*The velocity exponent n of equation (6) is of minor importance for any given meteor since the change in velocity along the trajectory is usually small.

$$\sigma = \frac{\Lambda}{2\Gamma\zeta} = \frac{I}{V^{n+1} \quad \dot{V} \int I/V^n \, dt} \tag{9}$$

and

$$K = 2^{-1/3} \Gamma \, A \tau_0^{1/3} \, \rho_m^{-2/3} = - \frac{\dot{V} \left(\int I/V^n \, dt \right)^{1/3}}{\rho \, V^2} \tag{10}$$

where for convenience we have now dropped the subscripts on m. The differential equations representing the meteor's motion are then

$$\dot{V} = K \left(\frac{2}{\tau_0} \right)^{1/3} \rho \, V^2 \, m^{-1/3} \tag{11}$$

and

$$\dot{m} = \sigma m V \dot{V} ,$$

so that

$$m_1 = m_2 \; e^{\sigma/2 \, (V_1^2 - V_2^2)} \quad . \tag{12}$$

The integrated form of equation (12) can be used to predict a terminal mass m_2 that remains after the initial mass m_1 has decelerated to a velocity V_2, where ablation ceases.

The success of the classical theory for even a limited number of objects recommends it as a point of departure for other problems. The quantity K has played a significant role in the physical theory of meteors. Independent determinations of the luminous efficiency τ_0 (Fe) of iron particles of known mass (McCrosky and Soberman, 1963; Friichtenicht et al., 1968; Ayers, 1965), together with an assumption on the iron abundances of meteoroids, now permit us to regard τ_0 as relatively well known. The shape factor A and the density ρ_m are now the least-known parameters of K. The observed values of K, interpreted under the classical single-body theory, are consistent with low-density spheres or very highly flattened bodies of meteoritic density. While it is true that this result has depended on an assumption of the composition of the meteoroid, some confirmation of the reasonableness of this assumption is provided by a few unusual meteors of abnormally low values of K (Cook et al., 1963; Verniani, 1966). In these cases, if a shape factor corresponding to a spherical meteoritic stone (ρ_m = 3.5 g/cm^2) is assumed, the luminous efficiency derived from the observations is in good agreement with the values deduced from the various iron-particle experiments.

Had the history of meteor physics proceeded along the lines we have described, it would seem that single-body theory and low-density meteoroids would have become de rigueur. In fact, the analysis of faint meteors (Jacchia, 1955) obtained with the Baker Super-Schmidt meteor cameras in the 1950s (before the question of the luminous efficiency was fully resolved) showed that outlandish departures from single-body theory were common in these objects. Two distinct approaches can be recognized in the ensuing attempts to reconstruct an adequate theory. One group, primarily the Harvard-Smithsonian contingent, proposed only to adjust the theory for small bodies and to accept as correct the apparent single-body behavior of the small-camera meteors. Another group, comprised of almost everyone else in the field, viewed the Super-Schmidt results as a possible symptom of a fatal error in the entire theory and consequently undertook the far more difficult task of rewriting the meteor theory in terms of new physical concepts. It has been a general practice in these attempts to introduce the simplifying assumption that meteoroids are similar in structure to meteorites. The need for some such assumption is understandable since the models depend on the bulk behavior of the material. Some authors state explicitly (and others seem to imply) that their models have additional merit because an explanation is made on the basis of a meteoritic structure. We consider this conclusion unwarranted. The question of the existence of another kind of meteoroid structure is too important to resolve by hypothesis. If meteoroids are in fact low-density material, their physical characteristics either cannot or need not be similar to those of meteorites. If either is the case, the high-density model is erroneous and must fail.

It appears that no group has sufficiently compelling arguments to convert its opponent. The stumbling block is invariably in the extrapolation of the faint-meteor explanations to the larger bodies observed by small cameras. In the following two sections, we will attempt to extrapolate these new models to the case of extremely large bodies and to demonstrate their failure by observations made on bright fireballs.

3. VARIATIONS IN THE SINGLE-BODY THEORY AND THEIR EFFECTS ON LARGE AND SMALL BODIES

3.1 Progressive Fragmentation

The nature of the so-called faint-meteor anomaly is adequately described elsewhere (Jacchia, 1955). In brief, it was found that the dynamic mass decreased far more rapidly than the photometric mass as the meteor progressed along its trajectory, giving the appearance of a decreasing meteoroid density with increasing time. Changes in the basic theory of a "mathematical," rather than a physical, nature cannot explain the anomaly. Any method that treats the entire body of data as a statistical sample rather than investigates the effects observed in an individual meteor may be subject to this objection because meteors of identical mass and velocity may produce greatly different anomalies. An adequate theory, therefore, must permit an uncertainty or variability in the physical process that produces the anomaly. Ananthakrishnan (1960, 1961) proposed that the anomaly be removed by assuming that the luminous efficiency depends on the atmospheric density. In this way, the "average" meteor can be made to obey the new theory, but most meteors will remain anomalous. Furthermore, bright meteors that previously followed the classical theory will, when analyzed under the revised theory, now show a strong anomaly of the opposite sense.

Jacchia's original suggestion of a progressive fragmentation of the meteoroid into an ever-increasing number of fragments explains well the deceleration anomaly. His results included a measure of the degree of progressive fragmentation in the index χ. Visible forms of fragmentation, such as terminal blending, are strongly correlated with χ. Jacchia proposed that the crumbling is primarily a surface phenomenon whereby small fragments, perhaps nearly commensurate with the fundamental "building-block" size in a porous structure, are detached. If the same kind of fragmentation occurs in large and small bodies, the effect on deceleration will be increasingly apparent as body size decreases. Large bodies will approach the classical behavior.

We are not aware of any serious discrepancies between this explanation and observations. Allen and Baldwin (1967) have questioned Jacchia's interpretation because of the occurrence of meteors with $\chi < 0$, "unfragmenting" meteors. As observers we are perhaps less distressed by these cases than are other investigators. We note, though, that $\chi < 0$ is the expected result of either a body that ablates to a more streamlined form or of a porous body that collapses during the melting process. Other causes of negative χ, appropriate for fireballs, are discussed in Section 4. On the other hand, we can foresee some difficulties with Jacchia's model if the surface of the meteoroid has time to increase its strength because of melting and subsequent freezing of molten material in the interstices.

3.2 Frothing and Sloughing

Allen and Baldwin's (1967) proposal that a deceptively low-density object is produced from a high-density source by forming a shell of solidified froth is particularly appealing since they have demonstrated the possibility by direct experiment. This model not only explains the apparent low density but also, if froth fragments from the body, predicts terminal blending, flaring, and all the other aspects of meteors usually attributed to fragmentation of a fragile body. The faint-meteor anomaly is produced by a steady increase in the froth. Increased vaporization and sloughing of froth in larger bodies are proposed to eliminate the anomaly in small-camera meteors.

Allen and Baldwin indicate that they encountered some difficulty in maintaining the integrity of the froth in their well-controlled laboratory experiments. Their inability to approach the dynamic or thermal loads met by a natural meteor still remains a major question in determining the applicability of their process to the meteor phenomena. In an example quoted in a second paper, Baldwin and Allen (1968) extrapolate from a test made on a 30-g sample with heating rates comparable to a 15-km/sec meteor at an altitude of 98 km to what might be expected of a 1-g meteoroid of the same velocity at an altitude of 86 km. They conclude that frothing sufficient to produce a significant decrease in the apparent density will then occur after 3.6 sec of heating. Of course, 1-g meteors do not have this long a lifetime. The average 15-km/sec meteor first becomes luminous at about 80 km or, for most zenith angles of the radiant, less than 1 sec after passing the 86-km level. It would appear that, if frothing is to be significant, most of the process must occur after the onset of luminosity and in a region where the dynamic load — and the thermal load — on the froth will be nearly 2 orders higher than those characteristic of the laboratory experiment. For many Prairie Network fireballs (height about 40 km), the aerodynamic loads are 2 more orders higher. These bodies may have ablated to a size where the frothing process may again play some role, but the environment must be extremely hostile to the frothing phenomenon.

Froth on meteorites is very rare. The exception is the Sputnik 4 steel fragment, which contains about 10 to 15% of its mass in porous iron droplets (Marvin, 1963). However, the low velocity and the tangential entry provided the long heating period and the low aerodynamic pressure that are conducive to accumulating frothing.

3.3 Fragmentation by Thermal Shock

Jones and Kaiser (1966) have suggested that thermal stresses in stony objects can, for sufficiently large bodies, produce major fracturing before other forms of ablation become appreciable. If this is the case, Jacchia's concept of progressive fragmentation in a weak structure is unnecessary. To invoke these thermal forces, the authors demonstrate that a strong thermal gradient will exist in meteors of radius $R \gtrsim 0.1$ cm. Assuming that the heating of the body proceeds as though the meteoroid were plunged into a hot bath, they show that the thermal stresses at the center of a spherical meteoroid will exceed the tensile strength and therefore, they presume, the body will fragment. However, their approach omits from consideration several factors of importance for bodies of $R >> 0.1$ cm. For example, the proposed heating mode ignores ablation and has associated with it a time constant that may be very large compared with the duration of a meteor event. The maximum stresses for stones of 10-cm radius are reached only after the object has been immersed in the bath for nearly a half hour! We show in the following sections that, in the regime of large R, thermal fracture should not be expected and, therefore, that fragmentation of this nature is also size-dependent and cannot be important for many Prairie Network objects if they are homogeneous stones. We will consider the thermal-shock problem first during the preheating period and then during the ablation period. Later in Section 3 we will investigate ablation by the spallation associated with thermal shock.

3.3.1 Thermal stresses during the preheating period of a meteoroid

We assume a spherical meteoroid of radius R composed of a homogeneous material. The thermal stresses in the solid material can be computed according to the formulas of Timoshenko and Goodier (1951, p. 418, formulas (252)):

$$\underline{P} = \frac{2\alpha E}{1-\nu}\left(\frac{1}{R^3}\int_0^R T r^2\,dr - \frac{1}{r^3}\int_0^r T r^2\,dr\right) \tag{13}$$

$$\Pi = \frac{\alpha E}{1-\nu}\left(\frac{2}{R^3}\int_0^R T r^2\,dr + \frac{1}{r^3}\int_0^r T r^2\,dr - T\right) . \tag{14}$$

The positive values of $\underline{P}$ and Π correspond to tension, and the negative values, to compression. If the temperature T is given as a known function of r, the integrals can be evaluated and the stresses computed. Here $\underline{P}$ is the stress along the radial direction and Π the stress in any direction perpendicular to the radius vector of the spherical meteoroid.

The problem of heating a meteoroid with $R \gtrsim 1$ cm during the first nonablation part of the trajectory can be solved if radiation cooling is neglected. This solution can be used for direct computation of $\underline{P}$ and Π. Only the starting equations and the solution are given here. The temperature of a meteoroid during the preheating period is given by the following set of equations:

$$\tau(t, r) = T - T_0 \quad , \tag{15}$$

$$\frac{\partial(\tau r)}{\partial t} - \beta^2 \frac{\partial^2(\tau r)}{\partial r^2} = 0 \quad , \tag{16}$$

$$\lambda \left(\frac{\partial T}{\partial r}\right)_{r = R} = \frac{\Lambda \rho V^3}{8} \quad , \tag{17}$$

$$\tau(-\infty, r) = 0 \quad , \tag{18}$$

$$\tau(t, 0) = \text{finite value} \quad . \tag{19}$$

The actual velocity V in equation (17) can be substituted directly by V_∞, the no-atmosphere velocity, because the change of velocity of such large bodies during the preheating period is negligible. Equation (17) contains a numerical factor 8, which represents the extreme assumption of omnidirectional heat flow to the surface of the meteoroid.

The solution of equations (15) through (19), derived by Ceplecha and Padevet (1961), is

$$\tau = \frac{R\,\tau(R)}{e^{WR} - e^{-WR}} \cdot \frac{e^{Wr} - e^{-Wr}}{r} \quad , \tag{20}$$

where

(21) $$W = \frac{(b \cos Z_R \, V_\infty)^{1/2}}{\beta} \quad ,$$

and

(22) $$\rho = \frac{8 \lambda \tau(R)}{\Lambda V_\infty^3 R} \left(WR \, \frac{e^{WR} + e^{-WR}}{e^{WR} - e^{-WR}} - 1 \right)$$

If we substitute equation (20) into equations (13) and (14), the resulting stresses are

(23) $$\underline{P}(r) = \frac{2 \alpha E \tau(R)}{1 - \nu} \; \frac{F(WR) - F(Wr)}{G(WR)} \quad ,$$

(24) $$\Pi(r) = \frac{\alpha E \, \tau(R)}{1 - \nu} \; \frac{2 \, R F(WR) + r \, F(WR) - RG(Wr)}{RG(WR)} \quad ,$$

where the functions F and G are defined as

(25) $$F(x) = \frac{x(e^x + e^{-x}) - (e^x - e^{-x})}{x^3} \quad ,$$

(26) $$G(x) = \frac{e^x - e^{-x}}{x}$$

For a meteoroid with given velocity and material constants, W is given by equation (21). Equation (22) then yields the air density ρ at which the surface temperature $\tau(R)$ is reached. If equation (21) for W is substituted in equations (23) and (24), we can compute the thermal stresses $\underline{P}$ and Π at any place in the meteoroid.

We now search for extreme values of stresses. Inspecting equation (23) we see that during the whole preheating period of a meteoroid $\underline{P}$ is always positive inside the body ($r < R$) and zero at $r = R$. It is also evident that the function $\underline{P}(r)$ decreases as r increases, and thus the maximal radial tension is always at the center of the meteoroid. If the body is sufficiently large, the tension is practically the same in the greater part of the interior and the drop to zero tension takes place in a thin surface shell.

On the other hand, if we inspect equation (24), the tangential stress Π can be both positive and negative inside the body. On the surface of the meteoroid, $\Pi(R)$ is always negative; i. e., tangential compression on the surface is always present. It is evident that the function $\Pi(r)$ decreases as r increases; thus the maximum tangential compression is always on the surface and maximum tangential tension is at the center of the meteoroid. It is also evident that for sufficiently large bodies, $\Pi(0) = \underline{P}(0)$.

Thus the extreme stresses of a meteoroid during the preheating period can be computed by substituting the limits on r in equations (23) and (24):

(27)

$$\text{Maximum tension} = \underline{P}(0) = \frac{2\alpha\, E\tau(R)}{1-\nu}\; \frac{F(WR) - 2/3}{G(WR)}$$

(28)

$$\text{Maximum compression} = \Pi(R) = -\frac{2\, E\, \tau(R)}{1-\nu}\; \frac{G(WR) - 3F(WR)}{G(WR)}$$

If we substitute into equations (27) and (28) the values of the tensile and the compressive strengths (S_t, S_c) of the meteoroid material, we can obtain the surface temperature at which the stress is relieved by surface or internal fracture (τ_c = critical temperature). At this point, some sort of fragmentation may be expected. When the surface of the meteoroid material reaches the compressive strength, the heating that follows will result in surface fracture, which may be accompanied by some spalling of small chips from the surface.

Surface cracks alone, without spallation, could also relieve the induced compressive forces. When the material reaches the tensile strength at its center, the heating that follows would cause an internal fracture, which could either immediately break the body into several pieces, or break it later when the cracks reach the surface by any sort of surface ablation, or weaken the body sufficiently for aerodynamic forces to complete the fracturing process.

Equations (27) and (28) were evaluated with $\underline{P}(0) = S_t$ and $\Pi(R) = S_c$, with stony and iron compositions of meteoroids of different radius R, with different velocities V (15, 30, and 60 km/sec), and with a choice of Λ (1, 0.1, 0.01) and cos Z_R (1, 0.1, 0.01). Some of the basic results are presented in Figs. 1a and 1b, where the resulting critical temperature τ_c is plotted against log R. The critical air density at which the material strength is reached is plotted against log R in Figs. 2a and 2b.

The numerical values, in cgs units, of the other parameters used are

$$\nu = 0.25, \quad b = 1.6 \times 10^{-6}, \quad T_0 = 280°K \quad ,$$

Stone: $\lambda = 3 \times 10^5, \quad \rho_m = 3.5, \quad c = 10^7 \quad ,$

$$\alpha = 5 \times 10^{-6}, \; E = 5 \times 10^{11}, \; S_t = 5 \times 10^7, \; S_c = 2 \times 10^9 \quad ;$$

Iron: $\lambda = 3 \times 10^6, \; \rho_m = 7.6, \; c = 7 \times 10^6 \; ,$

$$\alpha = 1.5 \times 10^{-5}, \; E = 1.7 \times 10^{12}, \; S_t = 8 \times 10^9, \; S_c = 1.5 \times 10^{10} \quad .$$

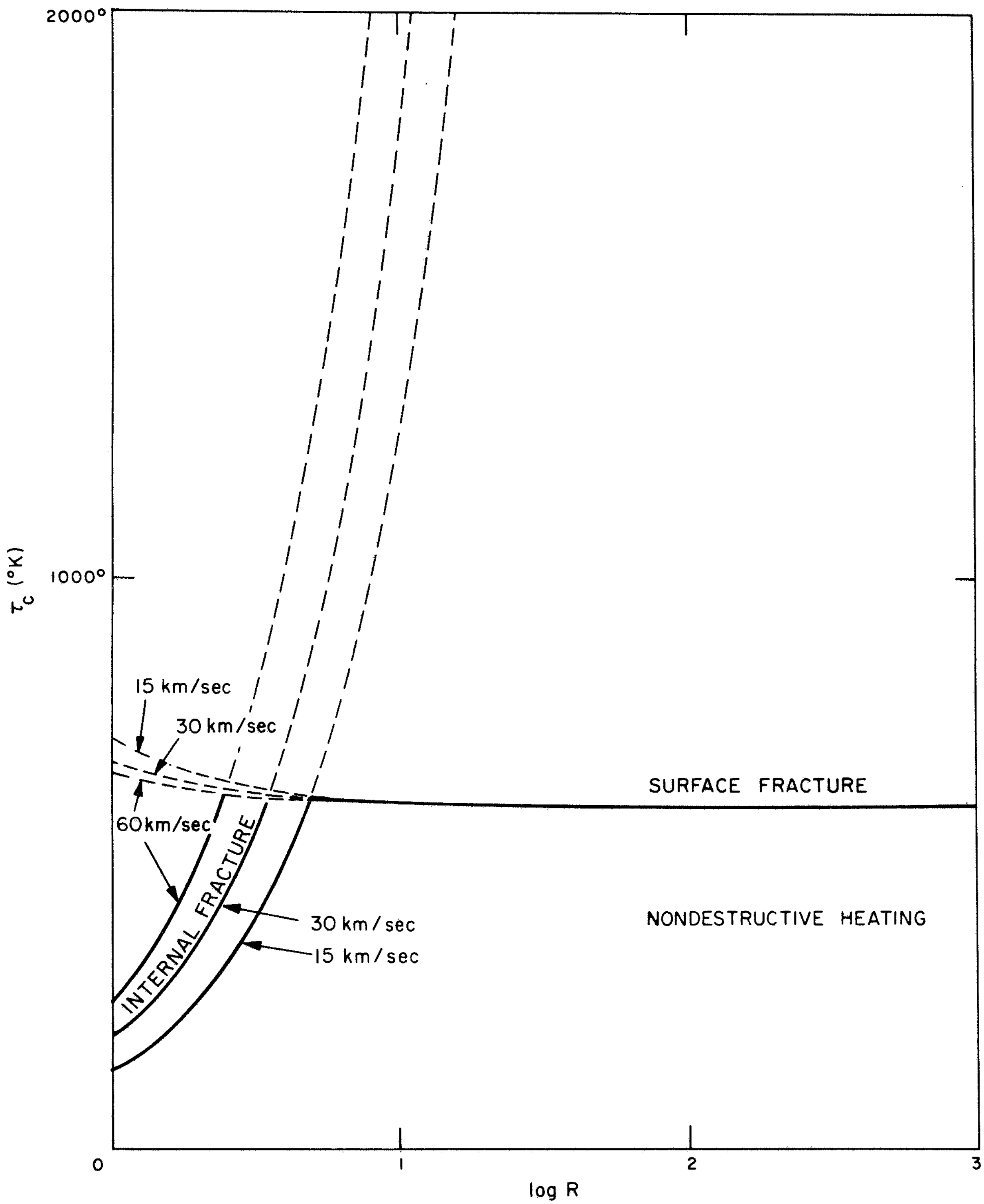

Fig. 1a. Meteoroid surface temperature at which the material strength is reached: Stone.

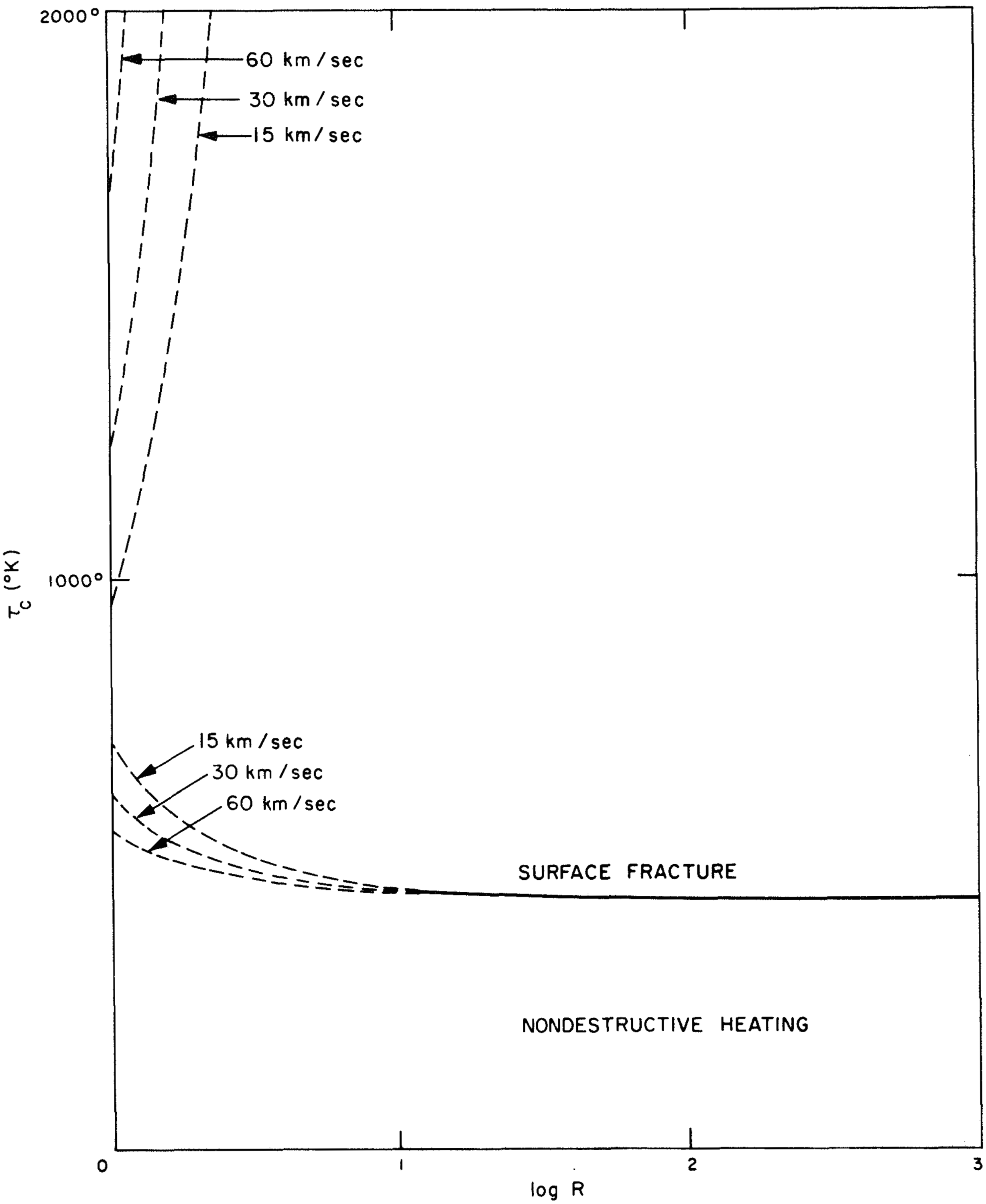

Fig. 1b. Meteoroid surface temperature at which the material strength is reached: Iron.

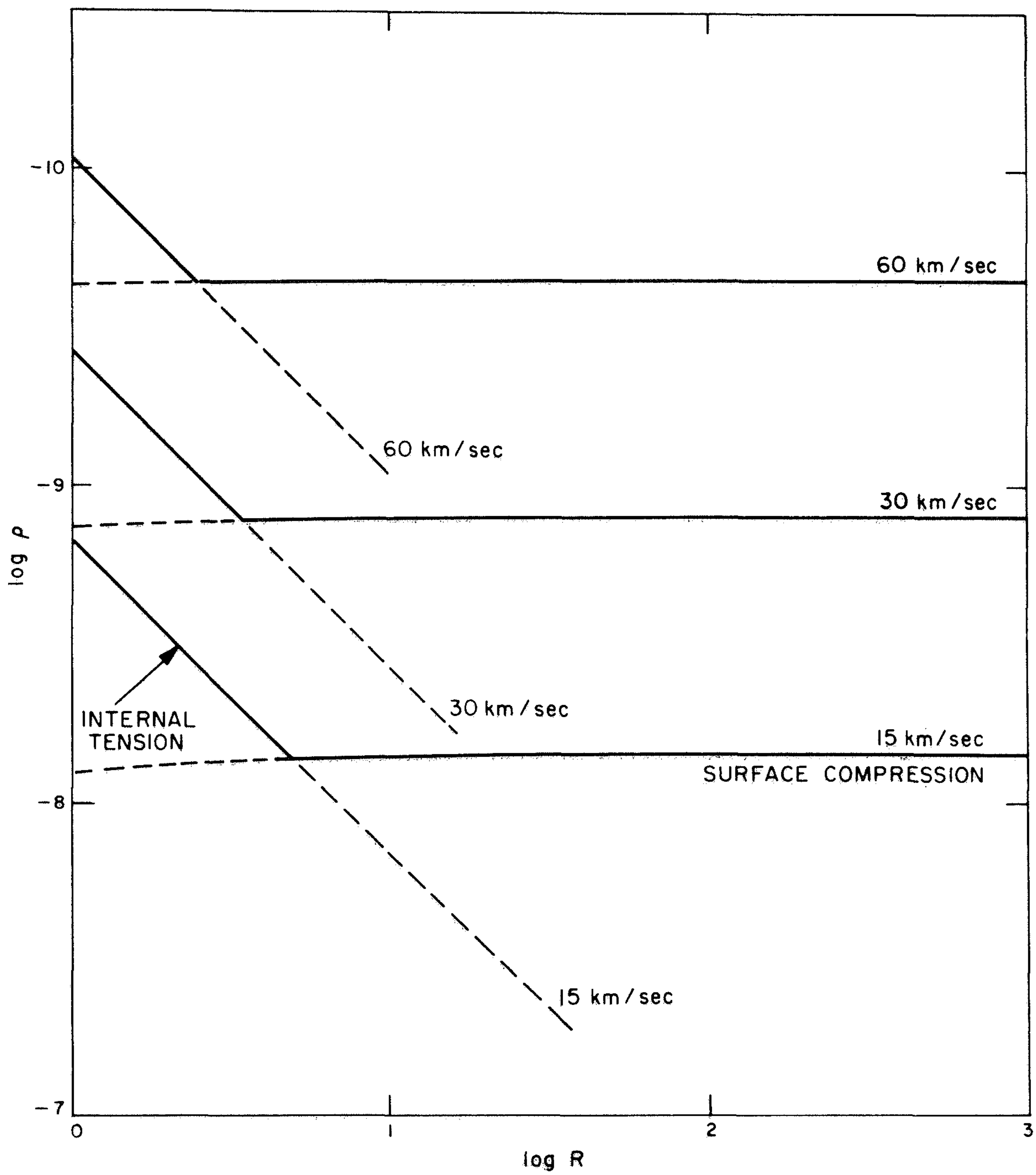

Fig. 2a. Air density where the strength of the material is reached: Stone.

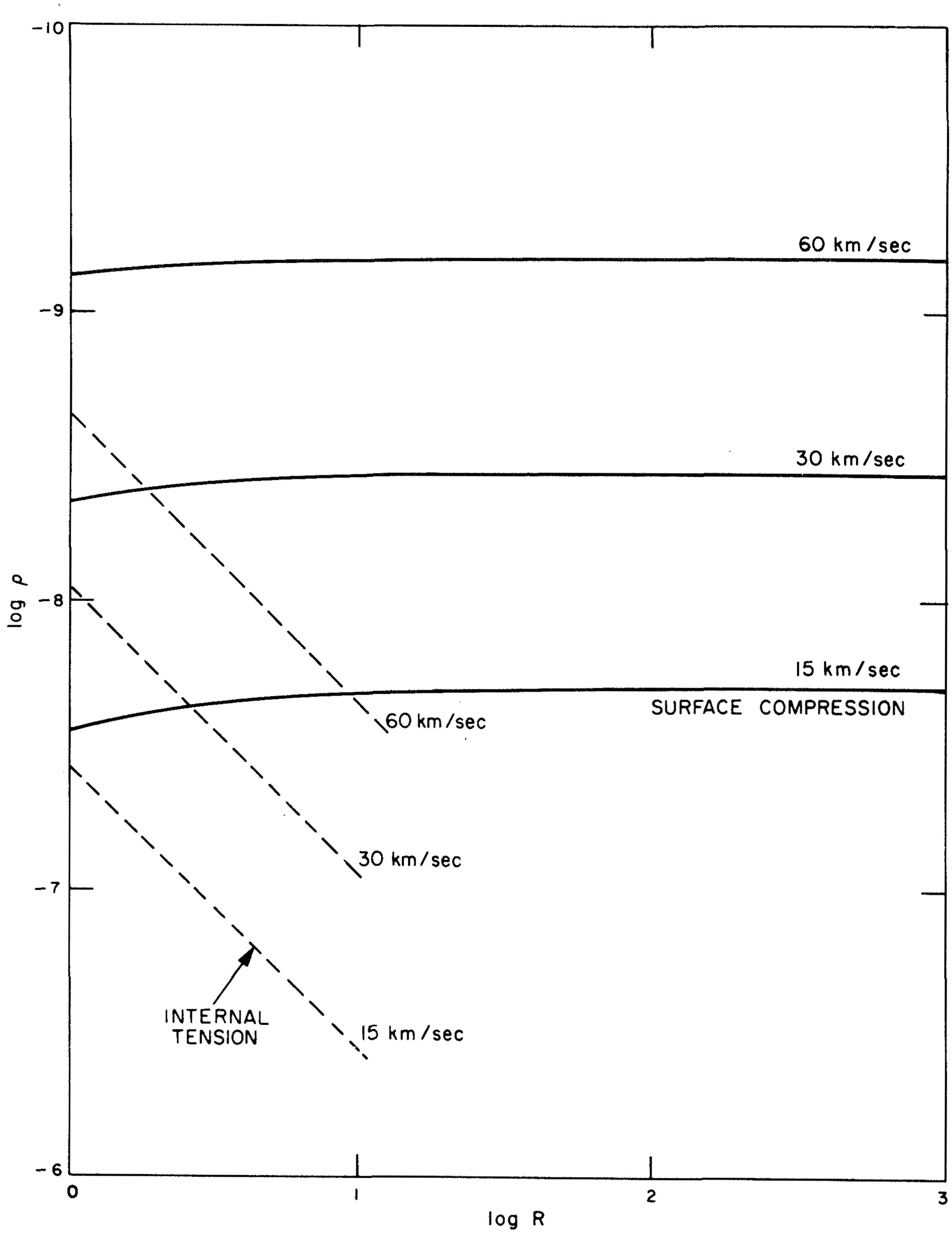

Fig. 2b. Air density where the strength of the material is reached: Iron.

Results. We can define a certain limiting radius R_{lim} of the spherical meteoroid for which the compressive stress at the surface and the tensile stress at the center are both equal to the corresponding strength of the material. For all $R > R_{lim}$, the strength of the material is always reached at the surface, which means that no internal fracture is possible. For all $R < R_{lim}$, the strength of the material is always reached at the center of the body.

The critical temperature τ_c can be defined as the temperature at which the material strength of the meteoroid is reached. For a stony meteoroid and $R > R_{lim}$, τ_c is almost constant ($\tau_c \doteq 600°$ C with the numerical parameters used). For a stony meteoroid and $R < R_{lim}$, τ_c decreases rapidly with decreasing R. This could be partially compensated for by radiation cooling, which had not been taken into account when the formulas were derived.

The critical temperature is dependent on only three material constants, α, E, and ν, for sufficiently large bodies ($R \gtrsim 5$ cm). The limiting radius R_{lim} depends on all material constants and, in addition, on V_∞ and $\cos Z_R$. The limiting radius is independent of Λ.

For stone,

$$\log R_{lim} = 3.78 - 0.5 \log V_\infty - 0.5 \log \cos Z_R \quad .$$

The critical air density ρ_c at which the critical temperature τ_c is reached depends on material constants and on V_∞, Λ, $\cos Z_R$, and b:

$$(29) \qquad \rho_c = \frac{8 \lambda \sqrt{b \cos Z_R}}{\Lambda \beta V_\infty^{2.5}} \tau_c$$

For stone,

$$\log \rho_c = 7.29 - 2.5 \log V_\infty - \log \Lambda + 0.5 \log \cos Z_R \quad ,$$

and for iron,

$$\log \rho_c = 7.75 - 2.5 \log V_\infty - \log \Lambda + 0.5 \log \cos Z_R \quad .$$

For all iron meteoroids considered here ($R \gtrsim 1$ cm), the tensile strength cannot be exceeded inside the body; internal fracture is impossible. The critical temperature τ_c for iron meteoroids with $R > 10$ cm is almost constant ($\tau_c = 440°$ C with the numerical parameters used). The critical temperature τ_c for an iron meteoroid increases as the radius decreases below 10 cm. For 1-cm radius, $\tau_c \doteq 720°C$ for $V_\infty = 15$ km/sec and $\tau_c \doteq 560°C$ for $V_\infty = 60$ km/sec.

3.3.2 Stresses during ablation period

If $R > 1$ cm, equations (23) and (24) reduce to

$$(30) \qquad \underline{P}(0) = \frac{2\alpha E}{1 - \nu} \frac{\tau(R)}{RW} = \Pi(0) \quad .$$

Using $T(R) = 1800°$ C, the temperature of the solid surface (i.e., $\tau(R) \approx 1500°$ C), and $W \approx 20$/cm, we find for the limiting radius at which internal fracturing may be expected,

$$(31) \qquad R_{lim} < 10 \text{ cm} \quad .$$

While unequal heating of a nonrotating body or excessive stresses caused by heating an irregular body may increase this limit, this simple calculation suggests that thermal fracturing is restricted to bodies of size between two limits, the lower limit being that determined for the isothermal case by Jones and Kaiser (1966).

When internal fracturing occurs, some strains are relieved and the nature of the problem changes. It is not obvious to us that the fracture planes will necessarily extend to the surfaces. Internal fracturing may not result in fragmentation. One might as well expect a shattered interior held together by an intact shell. This kind of model has been verified by a multitude of children who have engaged in the "fried marble" fad — glass marbles are subjected to sudden high temperatures that produce a decorative interior craze while the marble remains intact and smooth.

This result, of course, cannot be generalized to include material as inhomogeneous as chondrites, but we believe it is indicative of the process that may occur during the heating of meteorites. Meteorites that have their surfaces completely covered with a fusion crust are, of course, quite common. Some of these are very much smaller than $R_{lim} \approx 10$ cm. We suggest that such bodies do exist because the stresses were released by internal fracture.

3.4 Ablation by Fragmentation of Small Particles

If we wish to accept the meteor theory in detail, some revisions to the equations are necessary when fragmentation or droplet spraying is a significant method of ablation from the parent body. While the precise nature of the fragmentation process is not of vital concern to us, we can conceive of three possible mechanisms: hot material may be sprayed off as droplets; warm solid chips may be created by spallation; and cold or warm solid fragments may be detached by a pressure fragmentation. These three mechanisms represent cases where $\dot{m}_d \neq \dot{m}_v$.

In the last instance, the mass-loss equation (2) should be rewritten with a dependence on V^2 instead of V^3, thus introducing a second variation in the theory. Anticipating the result of this section — that small-particle ablation does not significantly affect the interpretation under the single-body theory — we now need only demonstrate that the velocity dependence in equation (2) either is unimportant or, at least, is not detectable in the observations. A set of equations comparable with those of the single-body theory is readily derived for mass loss by fragmentation. A new mass-loss parameter σ_f similar to σ but of different physical units, is found to be an observable quantity. Meteors generally show little change in velocity along the trajectory and therefore σ_f will show the same constancy as σ. The velocity dependence of these quantities among meteors of different velocities will, of course, differ and can, for example, lead to different predictions of the terminal masses of bodies of different velocities when equation (12) is used. It seems unlikely to us that cold fragmentation can occur throughout the trajectory, and although either form of mass-loss equation can be made to satisfy the observations of small-camera meteors, we believe the usual energy dependence given in equation (2) is more realistic.

As an example for detailed investigation of ablation by fragmentation, we consider a warm chip spalled by thermal shock. We will assume that all the energy of interaction of the meteoroid with the air is spent in spallation, and hence (1) the surface temperature will be constant during the ablation and equal to the critical temperature τ_c defined by equation (28); (2) the fragment is unshielded by the parent body; and (3) the fragment is sufficiently small so that further mass loss can occur only by vaporization — i. e., either tensile strength or surface tension precludes further fragmentation. We further suppose the following idealized history of the small chip after its release.

Initially, the fragment is heated through until it becomes a liquid at the vaporization temperature, during which time it is decelerated. The droplet is maintained at this temperature by radiation and ablation cooling. Mass loss and luminosity occur until the energy flux is balanced by radiation. Under these conditions, the terminal mass of the fragment is controlled by a small mass-loss parameter $\Lambda/2\Gamma\zeta$, since ζ is necessarily the vaporization energy and Λ, the effective heat-transfer coefficient, is less than the actual heat-transfer coefficient because of radiative cooling. Thus, we can expect the terminal mass, not accounted for in the photometric mass, to be larger than that determined by the application of equation (12) where the applicable value of ζ may be a fragmentation energy or the heat of fusion.

Furthermore, all the luminous energy will be produced by a fragment with a velocity less than that of the parent body. In the application of the luminosity equation (6), the body velocity and not the fragment velocity has generally been used. Thus, the photometric mass of the fragment will also be underestimated. To obtain some estimates of the errors in the photometric mass m_p resulting from the various effects, we will compare our result with an extreme form of ablation — direct vaporization from the surface of a meteoroid. A proper treatment of the problem would allow for differences in the heat-transfer coefficient Λ between a vaporizing body — with the attendant shielding — and a spalling body, and for differences in Λ between the large parent body and the small fragments. We ignore these effects and use $\Lambda = 1$ throughout. In so doing, we greatly exaggerate the spall rate and therefore have an upper limit to the discrepancy we would expect.

Ablation by spallation can be represented by the following differential equations:

$$\text{(32)} \qquad \frac{dR}{dt} = - \frac{\beta^2 W^2 R}{WR - 1} , \quad dm = 4\pi \rho_m R^2 dR ,$$

$$\text{(33)} \qquad \frac{d\rho}{dt} = b \rho V \cos Z_R ,$$

$$\text{(34)} \qquad \frac{dV}{dt} = - \frac{3\Gamma \rho V^2}{4 R \rho_m} ,$$

$$\text{(35)} \qquad W = \frac{\Lambda \rho V^3}{8\lambda \tau_c} + \frac{1}{R} .$$

For initial values ($t = 0$ at $\tau = \tau_c$), we take R = the initial radius, ρ_c, and V_c.

These equations were solved numerically for stony meteoroids of different R and V_∞. The mass loss corresponding to the decrease of R by dR during one integration step dt is assumed to be spalled in fragments of equal dimensions. The influence of nonregular distribution of the spalling velocity is neglected here. It is examined in the next section. The number of all fragments in one integration interval dt is computed from

$$(36) \qquad N = - \frac{3\, dm}{4\pi R_f^3 \rho_m} \quad ,$$

where the subscript f refers to a fragment or chip. We use $R_f = 0.01$ cm for $V_\infty = 15$ km/sec, 0.007 cm for 30 km/sec, and 0.005 cm for 60 km/sec, assuming that any larger chips would be fragmented by thermal stresses.

The initial temperature of each chip is assumed to be τ_c, and it is then heated to the boiling temperature. The heating is assumed to be isothermal for such small chips:

$$(37) \qquad \frac{cR_f}{3} \frac{d\tau_f}{dt} + \sigma_R (\tau_f + T_0)^4 = \frac{\Lambda \rho_f V_f^3}{8} \quad .$$

The velocity of the chip in this nonablation part of the chip trajectory is computed from

$$(38) \qquad V_f = V_{cc} \exp \left[- \frac{3(\rho_f - \rho_{cc})}{4 R_f \rho_m b \cos Z_R} \right] \quad ,$$

where V_{cc} and ρ_{cc} are, respectively, the velocity and air density at the point where the chip left the main body.

After this initial heating, the conventional single-body theory is used with $\zeta = 8 \times 10^{10}$ ergs/g to compute the evaporation, velocity, position in trajectory, and light intensity of each fragment. The radiation cooling is considered in the mass-loss equation. Multiplying the light intensity by the corresponding number of chips, we have the total light intensity i of the "cluster" of chips spalled during one integration interval dt:

$$i = \frac{\tau_0}{2} N \frac{dm_f}{dt} V_f^3 \quad . \tag{39}$$

Comparison of theoretical light curves of the same object ablating by 100% spallation and by 100% vaporization for different masses and velocities shows that

(A) The discrepancy between the photometric masses decreases as the velocity increases. It is negligible for meteor velocities in excess of 30 km/sec.

(B) The discrepancy decreases as the meteor mass increases and is less than 10% for meteors of $R \geq 30$ cm for all velocities considered.

(C) For the extreme cases studied ($r = 3$ cm, $V = 12$ km/sec), the photometric mass of the spallation case underestimates the true mass by 33%. As a result, the density would be overestimated by about 18%, an entirely trivial correction.

3.5 Gross Fragmentation and an Interpretation of Some Super-Schmidt Meteor Data

If the restriction that the meteoroid is a single body is removed, a simple model of the meteoritic process that follows the spirit of the classical theory can be derived to explain the observed low density even though the actual bulk density of the object is high. Let us consider a meteoroid that has fragmented into N separate objects, each of mass m and frontal area A. The observed drag will be that of one of the objects, whereas the photometric mass will equal the sum of the individual fragments. The new condition we impose is specified by

$$m_p = N\, m_d \quad , \tag{40}$$

and then

$$K' = N^{1/3}\, K \quad , \tag{41}$$

$$\rho'_m = \frac{\rho_m}{N^{1/2}} \quad , \tag{42}$$

where the primed quantities are those derived from the observations if the unprimed ones are the true values for each fragment.

This model is not restricted to cases of exact division of the parent body into N components. The various fragments may be somewhat different in size and not undergo appreciable differential deceleration. The relationships of equations (41) and (42) will still be valid although the parameter N will be less than the number of particles.

Before this model is accepted, we require that (1) a mechanism for producing the fragmentation exists and (2) some observations lend themselves to an interpretation by the model. We accept, for the moment, the Jones and Kaiser (1966) suggestion that thermal shock provides the mechanism and proceed to inspect certain JVB (Jacchia et al., 1967) Super-Schmidt meteor data.

The cases chosen are all those with a small progressive fragmentation index ($\chi \leq 0.1$) and for which at least four deceleration solutions are tabulated. There are 66 such meteors. The condition on χ limits the cases to those meteors that generally conform to the classical theory. Meteors with a large number of decelerations are usually those longer and better observed objects for which corrections to the "shutter flutter" are well determined. We use the weighted average

$$(43) \qquad \overline{\Delta \log \rho} = \frac{\sum p \, \Delta \log \rho}{\sum p} ,$$

where the required quantities are given in Table 1.2 of JVB. For $\tau_0 = 10^{-19}$ (cgs and 0th mag) and for $\Gamma A = 1.21$ (appropriate for a sphere in free molecular flow), we can show that

$$(44) \qquad \log K = \Delta \log \rho - 5.83 = -\frac{2}{3} \log \rho_m - 6.35.$$

The values are plotted against $\log V_\infty$ in Fig. 3.

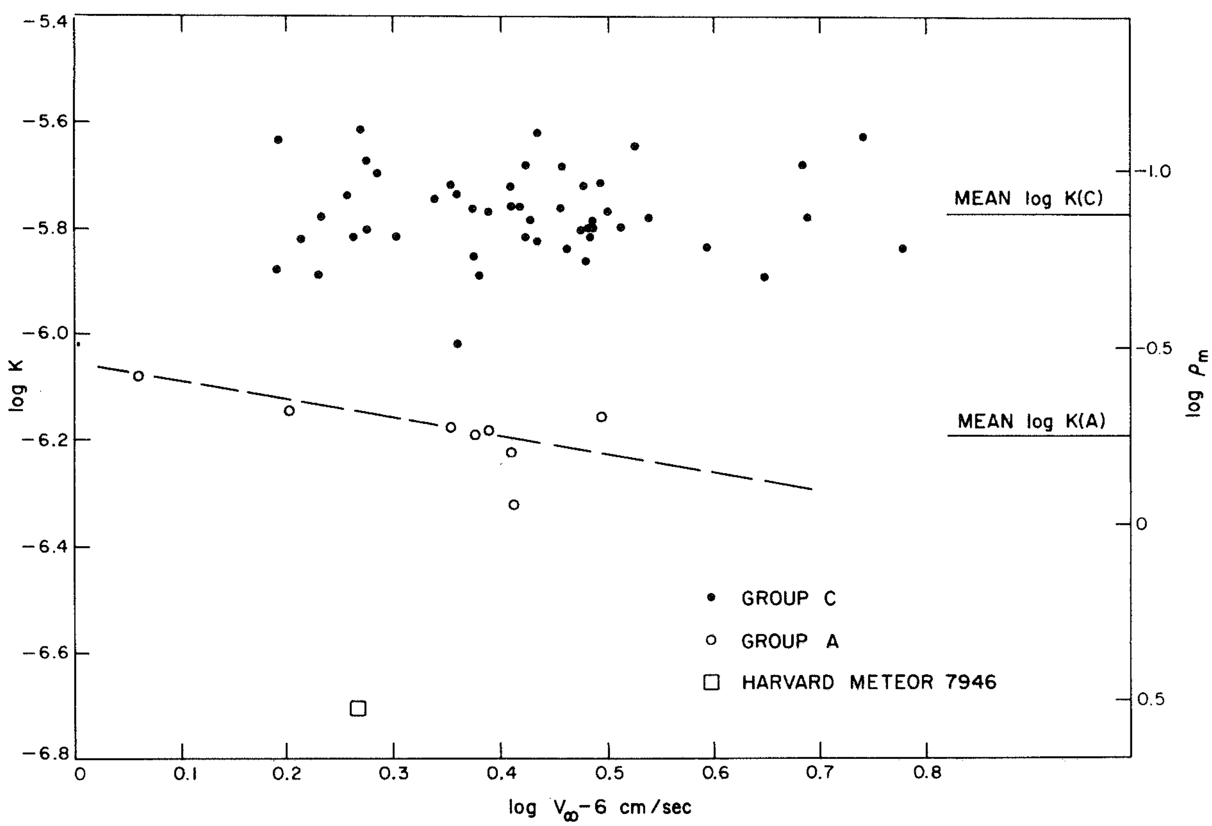

Fig. 3. JVB meteors with fragmentation index $\chi < 0.1$ separated into groups A and C according to Ceplecha's (1968) beginning-height criterion.

At first sight the observations suggest little more than a random scattering of the data. However, there is some indication that most of the meteors belong to two distinct groups. Other analyses of meteors from which this sample was chosen have also suggested that discrete differences in structure or density are to be found among these meteors. Jacchia (1963) divided the Super-Schmidt meteors into two groups according to their aphelion distance q'. Those with $q' > 7$ a. u. have statistically greater beginning heights than those with $q' < 7$ a. u. Using the same division of the data, Verniani (1965) demonstrated that the average value of K for these two groups shows a significant difference. We find, in our more limited sample, that 32% of the meteors with $\log K > -6.0$ have $q' > 7$ a. u., while none of the meteors with $\log K < -6.0$ has such a large aphelion distance. We can argue, then, that Fig. 3 is correctly interpreted as a composite of two groups and that aphelion distance is a useful, but not the complete, parameter required to separate them.

In an independent analysis, Ceplecha (1968) was able to separate the Super-Schmidt meteors solely on the basis of beginning height and demonstrate that selection effects in the JVB meteors artificially enhanced the importance of aphelion distance as a distinguishing characteristic of the two groups. We have used Ceplecha's criterion to separate the data into his groups A and C as shown in Fig. 3. (The stray point at $\log K \approx -6.7$ from Harvard meteor 7946 is the only "asteroidal" fragment uncovered by JVB. As such, it can be considered the unique member of a third group.) We believe this division into two groups has at least the same significance as the earlier analyses. Before we proceed with an interpretation of these meteors, it will be of value to discuss some observational errors.

We inspected the variations of log K and the associated values of $\log \sigma$ given by JVB for various deceleration solutions of <u>individual</u> meteors in group C. There is some tendency for points to scatter around a line

$$\text{(45)} \qquad \frac{\log K}{\log \sigma} \approx -1 \quad ,$$

a result to be expected if the most serious observational errors are in the deceleration. An error of about 20% in $\dot{V}$ is necessary to account for the scatter of log K around the mean value of group C. Jacchia has suggested that the internal probable errors of $\dot{V}$ are of the order of 5%.

Other possible errors and an estimate of the mean deviations they may produce in log K are 0.15 for actual differences in the meteoroid's shape or density, 0.05 for errors in the integrated intensity, and 0.05 for deviations of the actual atmosphere from the standard atmosphere employed in the analysis. Altogether it does not appear unreasonable to accept the scatter in the C group as observational in origin and take the mean value of log K = -5.77 ($\rho_m \approx 0.1$ g/cm^3) to represent the data. The average of the A group is log K = -6.19 ($\rho_m \approx 0.6$ g/cm^3). If the scatter is to be interpreted as resulting primarily from gross fragmentation and if the lower values of log K (-5.9 for group C) result from a single body, then the largest value is explained by a meteoroid fragmenting into about 10 equal masses or, alternatively, a still larger number of unequal but comparable masses. A division into five fragments for group A would suffice. These numbers are in the realm of possibility.

However, if we accept this interpretation, we must also accept the values of density implied by the extreme (single-body) value of K. As an alternative we can assume a meteoritic density $\rho_m = 3.5$ g/cm^3, but demand, in accordance with equation (42), that the variation in log K be caused by meteors with $12 < N < 60$ for group A (and $150 < N < 1500$ for group C!). We cannot believe this to be a serious possibility. First, it would be remarkable for all meteors to fragment. Second, terminal blending is seldom observed in these meteors with $\chi < 0.1$. It is inconceivable that the mass distribution of 25 fragments could be so uniform that different decelerations among them would go unnoticed.

An important by-product of this analysis is the suggestion that the apparent density ratio of the two types of common meteors is 4. In his first analysis of JVB meteors, Verniani (1965) suggested a density ratio of 1.4

for meteors separated by his aphelia criteria. We will use the densities derived above from the average values of log K in a later section. We defend them against Verniani's values with the following plausibility argument: Any method that maximizes a given variable that distinguishes two groups of objects suffers from less diffusion between the groups and, therefore, is to be preferred.

In his second paper Verniani (1966) described meteoroids of $q' < 5.4$ a.u. as having densities increasing with decreasing q'. Before obtaining this result, he removed a small part of his sample that had large deviations from the mean of log K. For the most part, small values of K (i.e., group A meteors) were discarded. Since these meteors have a statistically lower velocity than group C meteors (Ceplecha, 1968), they also have a lower q', and the effect of diminishing the sample was to decrease the apparent dependence of density on q'. The question then arises: Is the remaining dependence real or the result of an admixture of some group A meteors in the decreased sample Verniani used? In his final analysis of densities of all meteors, Verniani has shown that the distribution of densities is bimodal for meteors of low q'. The high-density group has a modal value comparable with our group A value. He argues that a constant shift toward higher density with decreasing q' for the meteors in the low-density mode is partly responsible for the density—aphelion relationship in his reduced sample. This is certainly correct, since the majority of high-density objects were not present in that sample. However, we cannot be certain that the observed relationship is not caused primarily by the increased proportion of A meteors still remaining in the reduced sample as q' decreases.

Figure 3 may also contain a seed of another important characteristic difference between A and C meteors. The best representation of the A meteors is given by $\log K \propto -1/3 \log V$ and is consistent with exponent $n = 2$ (instead of 3) in equation (10). There is no appreciable velocity dependence among the C meteors. The suggestion is at variance with Verniani's conclusion that neither of his groups contains a detectable velocity dependence, but as in the case of the density ratios, this could be an effect introduced by an inappropriate division of the data. Furthermore, the relatively small number of group A meteors in the JVB material would dilute the effect when the meteors are treated as a homogeneous group. Far better statistics will be required to demonstrate a convincing velocity dependence, but the consequences will be sufficiently important to recommend this as a future goal of any optical meteor program.

3.6 Spallation and the Reverse-Rocket Effect

The complete drag equation (4) initially proposed by Levin (1961) included, in addition to the normal aerodynamic force, a term resulting from the impulse of the vaporization products. If the mass loss $\dot{m}$ in equation (4) is given by equation (12), then equation (4) can be written as

$$(46) \qquad m\dot{V} = -\Gamma A \rho V^2 (1 + f w V \sigma^*) \quad ,$$

where $\sigma^* = \Lambda/2\Gamma\zeta$ will be referred to as the true value of the ablation coefficient. The observed value σ, if it is derived from a drag equation containing only the aerodynamic drag, is related to the true value by

$$(47) \qquad \sigma^* = \frac{\sigma}{1 - f w V \sigma}$$

As Levin pointed out, this reverse-rocket effect is important only for cases where the gas cap does not shield the departing vapors. This restriction, however, is not valid if the ablation particles are solid fragments rather than vapor. Here we will discuss the apparent changes in the drag forces induced by such spallation products if they should occur.

Let us define the effective drag coefficient as

$$(48) \qquad \Gamma^* = \Gamma (1 + f w V \sigma^*) \quad .$$

If we attempt to adjust the apparent density of the most dense group A meteoroid from the value of $\rho_m = 0.9\ g/cm^3$, the term in parentheses must take the value $(3.5/0.9)^{2/3} \approx 2.5$, or

$$(49) \qquad f w V \sigma^* \doteq 1.5 \quad .$$

Substituting σ^* from equation (47), utilizing an empirical value of σ for Super-Schmidt meteors (Cook, 1968),

$$(50) \qquad \sigma \doteq \frac{5.8 \times 10^{-6}}{V}$$

and solving for spallation velocity in the extreme case of f = 1, we find that spallation velocities of the order of 1 km/sec are required to give stony densities for meteors of group A.

It is of interest to investigate the fraction of the specific ablation energy ζ that is contained in the specific kinetic energy of the spall $w^2/2$. Certainly, $w^2/2 \leq \zeta$ and $\Lambda \leq 1$. Also, as shown by equation (47), $\sigma \leq \sigma^*$. It follows then that $w^2 \leq 1/\Gamma\sigma$. An extreme limit on $w \precsim 10^6$ cm/sec is found from the smallest observed values of σ ($2 \times 10^{-12}\ sec^2/cm^2$) among the Super-Schmidt meteors. A limit of $w \approx 3 \times 10^5$ cm/sec results from an average value of $\sigma \doteq 2 \times 10^{11}\ sec^2/cm^2$. In principle, there is sufficient energy available to explain the meteor bulk densities in terms of spallation, but the efficiency of the transfer of the available energy to the spallation process is, in our view, unbelievably high.

We can confirm this view by an investigation of a simple model. If we accept the concept of a strong solid body for the meteoroid, we can produce spall only if the material strength at the surface is exceeded. The maximum energy available will be that stored in the material by elastic compression at the failure point. Let us consider the compression of a cylinder of frontal area $\mathcal{A}$ and length L. When compressed by an amount ΔL, the work done is given by

$$U = \int_0^{\Delta L} f \; dL \quad . \tag{51}$$

Given Young's modulus $Y = PL/\Delta L$, where P is the applied pressure and will later be taken as the compressive strength of the meteoritic stone, we find that

$$U = \frac{1}{2} \frac{P^2 A L}{Y} = \frac{1}{2} \frac{m}{\rho_m} \frac{P^2}{Y} \quad ,$$

where ρ_m is the bulk density. Equating the work per unit mass U/m with a kinetic energy $w^2/2$, we have

$$w = P(\rho_m Y)^{-1/2} \approx 10^3 \text{ cm/sec} \tag{52}$$

for meteoritic stone. This is the maximum velocity of spall and is 2 orders less than required. The correction to the drag coefficient given by equation (48) is only of the order of 1%.

4. PHOTOMETRIC AND DYNAMIC MASSES OF FIREBALLS AND FAINT METEORS

A statistically significant number of very bright meteors ($-15 < M < -5$) have been observed by the Prairie Network (P-N) system (McCrosky and Boeschenstein, 1965). The data-reduction methods and detailed results for 29 of these have been published (McCrosky and Posen, 1968). Photometry and trajectory data for 150 such objects now exist. The P-N trajectory data are seldom of the high quality possible with Super-Schmidts. The poorer optics and the greater range of the object diminish the accuracy. However, the longer duration of these bright objects ($1 < t$ (sec) < 14) often permits us to determine decelerations with an internal accuracy comparable to that obtained for faint-meteor data. We most frequently use the observed trajectory data over a 2-sec interval to determine the deceleration at the midpoint of this trajectory arc. Since intervals overlap, the various deceleration values are not entirely independent. For example, a 6-sec meteor would normally be divided to give five solutions from trajectory arcs of 0 to 2 sec, 1 to 3 sec, 2 to 4 sec, etc. Those cases where the internal probable error of the deceleration is less than 12.5% of that quantity are discussed in this section.

Photometry of the bright P-N meteors also presents problems not encountered in faint meteors. In most cases, we have determined photometric masses from only the best photographs and have used these data for comparison with the dynamic masses determined from all films reduced for a particular meteor.

All these data are combined in Fig. 4, where we have plotted $m_p/m_d \propto \rho_m^2$ against m_p. For these meteors we have chosen $\Gamma = 0.46$, $A = 1.21$, and an arbitrary value of $\rho_m = 1$ in determining m_d. Photometric masses were derived from equation (7) with $n = 3$ and $\tau_0 = 10^{-19}$ (cgs and 0 mag). The same information is given for the 413 JVB

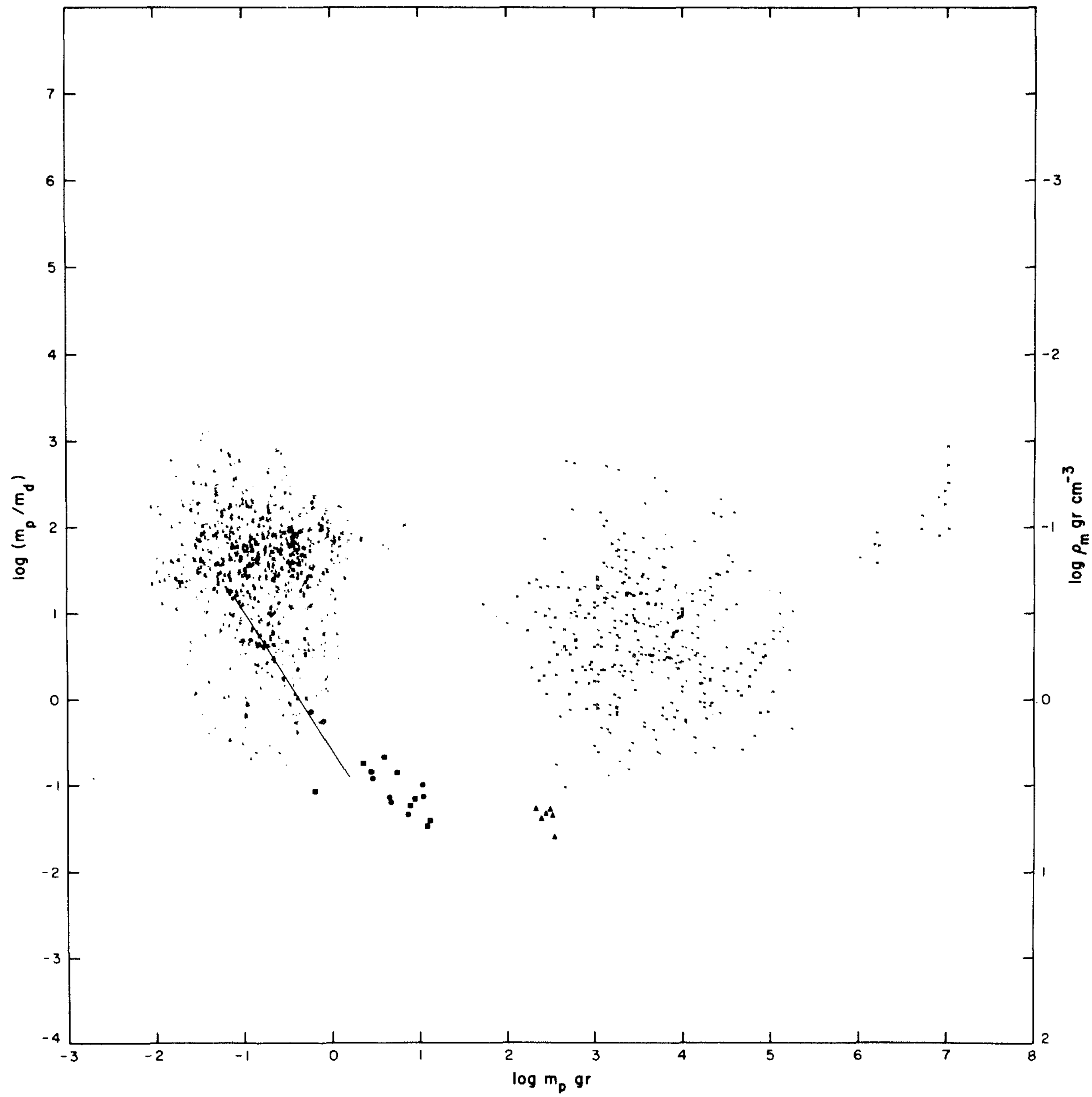

Fig. 4. Ratio of photometric mass (m_p) to dynamic mass (m_d), and density (ρ_m) as a function of m_p for JVB and P-N meteors. S = Super-Schmidt (JVB), N = Prairie Network, ▲ = Harvard meteor 1242, ■ = Harvard meteor 19816, and ● = Harvard meteor 7946 (JVB).

meteors for which the probable error of the deceleration meets the same condition as that required of the P-N data. For these smaller objects we have used $\Gamma = 1$ as a drag coefficient characteristic of free molecular flow. Each plotted point represents one deceleration solution, and thus each meteor may contribute many data points. The solid line within the body of data of the faint meteors represents a single hypothetical meteor with the average value of $\chi = 0.3$. The extensive vertical spread of these points, covering nearly 10 orders of magnitude, is in part understood by the behavior of meteors undergoing this degree of progressive fragmentation. The data points marked by special symbols are from three "asteroidal" fragments analyzed by Cook et al. (1963) or by JVB.

The extreme point at the upper edge of the diagram is a Giacobinid meteor. Meteors of this shower are unusual in every respect, and it is quite impossible now to determine how much of the anomaly shown in Fig. 4 is the result of fragmentation, intrinsic low density, or real departures from the usual meteor theory. Until recently, the abnormally great beginning heights of the Giacobinids were inexplicable. It appeared that radiational cooling would keep the material well below the vaporization point at these altitudes. Yet these meteors show the usual emission-line spectrum of iron. Cook's (1968) discussion of the radiation properties of particles of size comparable to the wavelength of the emitted light offers a mechanism for vaporizing material with a relatively low heat flux to the body. Giacobinids frequently show visible evidence of gross fragmentation at high altitudes. If they are also fragmenting appreciable quantities of micron-sized pieces at altitudes above 100 km, where the dynamic load is less than 4×10^2 dyn/cm^2, perhaps it is not so difficult to believe that their structure is little more than a gossamer.

The other extremely high and low values of the mass ratio of both the Super-Schmidt and the P-N results perhaps should be attributed to systematic errors of measurement. Other possible explanations are extreme shapes or an admixture of high-density material. There is no tendency for objects of initial mass larger than the critical size for thermal shock ($R > 10$ cm, $m > 15$ kg) to produce low mass ratios. Indeed, the opposite effect may be detected in the limited data of meteors of very large mass.

A least-squares solution for χ has been derived from those fireballs with at least three observed decelerations, the equation of condition being

$$(53) \qquad \frac{1}{3} \log \frac{m_p}{m_d} = \chi \log \left(\frac{m_\infty}{m_p} - 1 \right) + \text{constant}$$

The values of dynamic mass measured on all photographs of the meteor were combined in the solution. Eleven cases were rejected because of unusually large probable errors in χ (rms deviation > 0.25), as were all cases (25) when only two deceleration measures were available. The latter category consists primarily of short meteors for which the fragmentation index may be overly sensitive to small errors in the observations. The number distribution of χ for the remaining fireballs is compared in Fig. 5 with that given by JVB for small-camera meteors. The two distributions are similar except for a slight shift of the maximum of the fireball distribution to negative values of χ. A progressive decrease in the shape factor brought about by a streamlining of the ablating meteoroid or a terminal mass, unaccounted for in the photometric mass, can produce these negative values of χ. In either case, or in the cases discussed below, the corrections we would require to remove the skewness of the χ distribution will produce a <u>decrease</u> in the bulk densities implied by the observations.

The terminal mass is usually described as that part of the meteoroid remaining after all ablation ceases. In the case of fireballs in the P-N, where large ranges and great zenith distances are frequent at the end of the photographed trajectory, we must first rule out the possibility that the true terminal mass is not artificially enhanced by our inability to record the meteor during the last part of its trajectory. We examined the values of χ for those meteors with terminal velocities less than 8 km/sec in the hope that this selection would give a fair representation of meteors sufficiently near a station to permit them to be observed to near the end of their luminous trajectory. Of the nine meteors in this category, eight have $\chi < 0$. This fact denies the proposed explanation of the observed negative χ's and at the same time revives another possibility. If the luminous efficiency decreases more rapidly than the first power of velocity at very low velocities, the photometric mass will be an underestimate of the true mass. The suggestion that the luminosity law may fail at $V \approx 10$ km/sec has been made by a number of previous investigators (Öpik, 1958; Jacchia, 1949). With so many possible explanations, each of which may contribute to an error in the photometric mass, we do not think it fruitful to attempt to derive an expression for the luminous efficiency as a function of velocity capable of correcting the skewness of the distribution of χ.

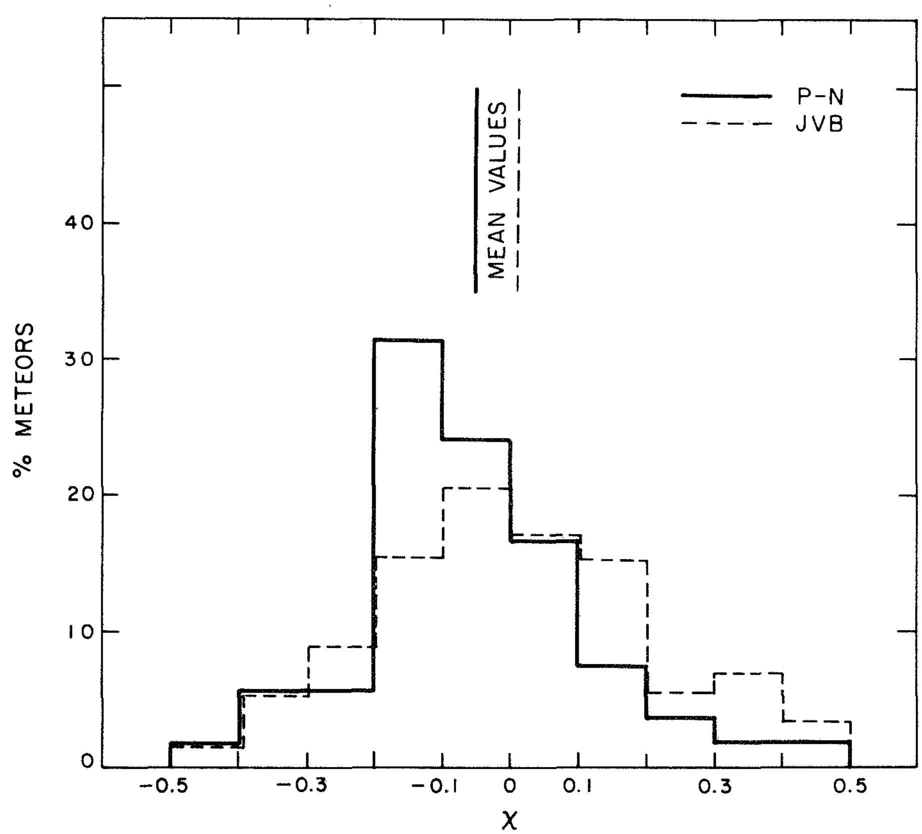

Fig. 5. Distributions of the fragmentation index χ for P-N and JVB small-camera meteors.

A primary feature of Fig. 4 is the general agreement in apparent density of many of the P-N objects with the values determined from the Super-Schmidts by Verniani or by us in Section 3.

The A and C groups cannot be distinguished among the faint meteors in Fig. 4, since meteors of both groups are strongly diffused along the ordinate by fragmentation. A separation of the fireballs into two groups on the basis of beginning heights is unreliable because of varying conditions of visibility during the observations. Ceplecha (1968) has suggested certain orbital criteria that would relegate almost all P-N meteors to group A or to subgroup C_1.

We believe the existing data are sufficient to provide reasonable upper limits on the density of the group A faint meteors and the average fireball. We immediately stipulate the possibility that group A objects may be comprised of relatively high-density material. The nominal density of 0.6 g/cm^3 can be increased by a factor of 2 if, for example, we accept an increase in the shape factor and luminous efficiency to the values given by Cook et al. (1963) ($A = 1.8$, $\tau_0 = 1.2 \times 10^{-19}$). If we arbitrarily change any of the constants to increase the density of group A meteors by more than a factor of 3, the densities of the various asteroidal meteoroids become anomalously high. These are all well-observed meteors with distinctive characteristics, and the results from these should not be discounted. One might be tempted to assign these objects to the iron-meteorite class to effect an increase in density of all Super-Schmidt meteors. But at the same time, some adjustment in the value of τ_0 is required because of the change in composition, and by a curious coincidence, stones and irons probably cannot be distinguished from one another by the observed value of K only. If stones contain 25% iron and if iron is the predominant contributor to luminosity, then

$$(54) \qquad K^2 \propto \frac{\tau_0}{\rho_m^2} = \left[\frac{\tau_0}{(3.7)^2}\right]_{\text{stone}} \approx \left[\frac{4\tau_0}{(7.8)^2}\right]_{\text{iron}}$$

If the densities are in error by a factor of 2 or 3, Whipple's (1967) pre-Type I carbonaceous chondrite or "half-baked" asteroid may describe a material of appropriate density and fragility to satisfy the observations of group A meteors. We note, however, that the density we derive for group A resulted from an analysis of a selected subgroup with relatively little fragmentation. These particular meteors may be the extreme examples of high strength and density.

Figure 4 makes it most reasonable to assume that the fireballs are a mixture of the A and C material observed by the Super-Schmidts. However, if we take the view that the faint meteors are truly high-density objects and have their true structure masked by frothing or fragmentation by thermal shock, then either the large fireballs are a different kind of object of low density or the constants Γ, A, and τ_0 used in determining the fireball densities are erroneous. If we require a 10-fold increase in the predicted density of $\rho_m \approx 0.3$, then either (1) the drag coefficient for large bodies is $\Gamma = (10^{2/3})\ (0.46) = 2.1$; (2) the shape factor for the average large body is $A = (10^{2/3})(1.21) = 5.6$, corresponding, for example, to a cylinder whose height is less than 10% of its diameter; or (3) the luminous efficiency τ_0 has been underestimated by a factor of 100.

We would, at the very least, be presumptuous if we attempted to redetermine the drag coefficient on the basis of an assumed meteor density. With respect to (2), highly flattened shapes without initial rotation will probably align themselves in flight with the flat face forward. But we have no reason to expect such extreme configurations. Isotropic etching by dust particles in space will tend to flatten any initially nonspherical body, but the initial shape must also be considerably flattened to produce the required shape with any tolerable etching rate.

The question of the existence of high-density meteoroids then rests primarily on the value of the luminous efficiency. Two problems are involved: Are the experimental values correct for the luminous efficiency of iron? Is the extrapolation of these values to meteoroid material appropriate?

The agreement of all the data cited previously makes it most difficult to disbelieve the general result obtained from the luminous efficiency of iron. Many additional artificial meteors in the atmosphere (Ayers et al., 1970) have confirmed the earlier Trailblazer results. Since the photometric measures can hardly be wrong by a factor of 100, any major discrepancies must be attributed to an erroneous value of the mass that entered into the meteor process. While it is true that the mass of a pellet is known only by ground studies of similar accelerating devices, the duration of the artificial meteor gives a rough measure of the mass of the flight model. If the true mass of the Trailblazer pellet is 10% (1% is required to explain away the low-density meteors!) of the nominal mass of 2.2 g, or if only 10% of the initial mass is vaporized, the velocity history of the observed pellet cannot be explained.

Allen and Baldwin made two attempts to derive a luminous efficiency for meteors that would justify their contention that small-camera meteors, for which the frothing process is less efficient, are also of high density. A partial explanation, in their 1967 paper, depends on blackbody radiation from carbon particles. The contribution of this luminosity is dependent on the abundance of carbon, the radiation temperature, and the sensitivity of the detection system to this radiation. Their upper limit of the suggested change in luminous efficiency is overestimated by at least a factor of 2 since they assumed a sensitivity of the optical systems to include the range 3000 Å to 7000 Å. The actual systems employed in obtaining the data they analyzed covered only the range 3800 Å to 5000 Å.

In the second paper, Baldwin and Allen (1968) again derived the luminous efficiency as a function of velocity on the assumption of high density and frothing meteoroids. To substantiate their results, a comparison was made with the laboratory results of Friichtenicht et al. (1968). We object to this comparison on two points. First, some of the increase of luminous efficiency with decreasing velocity that they attribute to Friichtenicht et al. is present only if one obviously spurious data point in the laboratory results is accepted.

Their plot of luminous efficiency as a function of velocity suggests $\tau_0 \propto V^{-1.7}$, whereas the experimenters themselves have used $\tau_0 \propto V^{-1}$ to express their results. Second, we believe the experimental data support rather than refute the constancy of τ_0 for meteors. Since elements other than iron become more important radiators as the velocity increases, the luminous efficiency for meteoritic material as a whole must increase with velocity more rapidly than that for iron.

While it may be possible to adjust the frothing model to accommodate these factors as they relate to fainter meteors, we cannot, for the reasons stated previously, conceive of frothing playing any appreciable role in the case of the fireballs. If they are of high density, the luminous efficiency must be increased 100-fold for large bodies. Furthermore, the increase of luminous efficiency with increasing mass would have to occur so as just to compensate for the decrease in the importance of frothing.

We cannot state, with finality, that such remarkable changes do not occur, but we can give qualitative evidence that suggests their extreme unlikelihood. Ceplecha (unpublished) has recently obtained a spectrum (56 Å/mm and 28 Å/mm) of a meteor estimated to be -18 mag, the brightest spectrum on record and equal to the brightest P-N object. The velocity was 18 km/sec at the beginning of the trajectory and about 6 km/sec in the final portion. Only minor differences exist between this spectrum and more usual objects of this velocity (faint Si II lines are detectable, for example). Iron is still the predominant radiator in the fireball, and this element alone must absorb nearly the entire increase necessary in τ_0. If we accept the value of Friichtenicht et al. for the absolute efficiency of iron in the range 3400 Å to 6300 Å,

$$\frac{\text{luminous energy}}{\text{kinetic energy}} \approx 0.5\% \quad ,$$

then nearly 50% efficiency <u>in the visual range</u> will be required. Until some mechanism for this efficient radiation is described, we prefer to accept the concept of low-density meteoroids.

5. METEORITES AND TERMINAL MASSES OF FIREBALLS

No meteorites have been recovered by the P-N system during 5 years of operation (see McCrosky and Ceplecha, 1969, for a discussion of the expectations and results of this operation). Extensive searches have been made for two objects that, if of high density, would have produced meteorites of less than 2 kg. The significance of this result is not that no recoveries have been made but rather that there have been so few objects that warranted a search. If the predicted rate of fall (one meteorite per year of greater than 1 kg) is correct, then the probability of 0 or 2 occurring in 5 years is 0.007 or 0.08, respectively. This unlikely dearth of meteorites prompts us to question (1) the observations or theories that produce these results, and (2) the assumptions that led to the predicted rate.

The determination of an upper limit to the terminal mass is one of the least sophisticated aspects of the reductions of our observations. Questions of the true luminous efficiency or of the ablation process are not involved in the problem. The distance D along the trajectory as a function of time is determined by the usual meteor-reduction procedures. We find that these data for most meteors with small terminal velocities ($4 < V < 8$) and low heights ($20 < H < 35$) can be adequately expressed by

$$D = a + bt - \frac{c}{2} t^2 \quad . \tag{55}$$

This expression, implying a constant deceleration c, is valid for the final 0.5 to 1.0 sec of the trajectory. The constants are determined from a least-squares fit. The internal probable errors are usually

$$\Delta b \approx 0.01b \quad , \quad \Delta c \lesssim 0.1c \quad . \tag{56}$$

With V and $\dot{V}$ given by the differentiation of equation (55) and with the standard atmospheric density, we can compute the dynamic mass at the end of the visible trajectory with equation (1). This will be an upper limit to the terminal mass since some additional ablation may occur. These masses rarely exceed 100 g for P-N meteors if we assume meteoritic densities and spherical shapes. There are, in fact, too few cases of $m_t > 100$ g for a significant statistical analysis, but we estimate that an error of at least a factor of 10, and probably 25, in the masses given by equation (1) is required to explain the apparent low rate of meteorites. Large systematic errors in Γ, ρ, and V are improbable. We have, perhaps, underestimated the shape factor. A cursory inspection of meteorite samples in museums suggests that an increase in A by a factor greater than $4^{1/3}$ (corresponding to a hemisphere in the maximum drag orientation) is unlikely. The final orientation of some meteorites can be distinguished by the fusion crust. A study of the shape factors of these objects could help define the average value of A and its dispersion.

Systematic errors in $\dot{V}$ can arise from errors either in D or t. The time scale is controlled by the frequency of a commercial power source. The errors in $\dot{V}$ due to a constant but abnormal power-line frequency are exactly compensated by errors in V^2; i. e., the dynamic mass is independent of the time scale. Variations in the line frequency over the 1 sec of the trajectory under consideration are small, and in any case, not of a systematic nature.

In determining D, we assume that the center of light of the trail also represents the center of mass of the meteoroid. If for any reason there is a progressive increase in the distance between the body and the light source with time, we would underestimate $\dot{V}$, and to a lesser extent, V^2. Let us assume as an example that the light phenomenon obeys equation (55) while the body follows the relationship

$$D' = a' + b't - \frac{c'}{2} t \tag{57}$$

We take $a' = a = 0$ and $b' = b + \epsilon$ without loss of generality. If we require a factor of 2 error in $\dot{V}$ from this effect, then $c' = c/2$ and

$$\Delta D = D' - D = \epsilon t + \frac{c}{4} t^2 > \frac{c}{4} t^2 \tag{58}$$

Measured values of c are of the order of 2×10^5 cm/sec^2. For t = 0.5 sec, $\Delta D > 125$ m, and for t = 1 sec, $\Delta D > 500$ m. The occulting shutter on the P-N camera operates at 20 cps. If the terminal velocity is 4 km/sec, the meteoroid, or the center of light, will travel 200 m during one shutter cycle, a distance comparable to ΔD. If the luminosity is thought of as a long wake behind the body with a maximum at ΔD, this model would predict a considerable smearing of the meteor image that would be evident as luminosity in the shutter breaks. Terminal portions of the trajectory often show no signs of this luminosity, and when wake luminosity is apparent it is frequently due to an obvious fragmentation process that has produced a small number of discernible pieces.

In summary, we do not believe it possible to modify significantly any of the quantities used to determine a terminal mass. We conclude from the fireball observations that meteorites in the range 1 to 10 kg do not reach the earth with the frequency expected and that we erred in our predictions.

It is well known that recoveries of individual small stones ($m < 5$ kg) are substantially less than would be predicted by a simple linear extrapolation of the distribution of bodies $m > 50$ kg. Brown's (1960) discussion of stone meteorite falls, which we used in our prediction process, suggests that meteorites of mass 1 kg are underabundant by a factor of 10 compared with the number expected from the extrapolation. Brown suggests that atmospheric attrition, in addition to the obvious selection effect, might be responsible for this decrease of numbers of small meteors. We would now agree with this conclusion and suggest that thermal fracture may be the important mechanism for the attrition of meteorites of initial mass $1 < m_0$ (kg) < 15.

Objects in the range $10 < m_0$ (kg) < 100 may survive with significant terminal mass. If they are structurally strong, their mass-loss ratio will be determined by the heat-transfer coefficient, which in turn may depend on the body size. If convective heating is important, large bodies will better survive; small bodies have the advantage if radiation transfer from the gas cap predominates. However, the objects at the low end of this range may ablate to sizes where thermal shock is again effective. Thus, there might be a tendency to increase the relative number of the larger meteorites derived from this class of objects.

Pressure fracturing in meteorites can be the primary attrition mechanism for those still larger bodies possessing sufficient momentum to penetrate deep into the atmosphere. No stones of mass greater than about 1000 kg have survived. Larger meteorites fragment and produce showers of relatively small stones. In the special case of a body entering an exponential atmosphere without mass loss, the maximum deceleration is reached when

$$V^* = V_\infty e^{-1/2} \quad , \tag{59}$$

and, by application of equation (1), when the atmospheric density is

$$\rho^* = \frac{m}{A} \frac{b \cos Z_R}{\Gamma} \ln \left(\frac{V_\infty}{V^*} \right) \tag{60}$$

The maximum stagnation pressure is then

$$(\Gamma \rho V^2)^* = \frac{m}{2A} \frac{b \cos Z_R}{e} V_\infty^2 \tag{61}$$

This maximum load is sometimes equated to the crushing strength S_c of the material. Thus, Öpik (1958, p. 26) has determined a strength of $S_c \approx 2 \times 10^8$ dyn/cm^2 for meteoritic stone, based on the heights of breakup of observed falls. This value is an order of magnitude below the uniaxial compressive strength of many terrestrial rocks and some meteoritic samples (Buddhue, 1942). Öpik resolves the discrepancy by suggesting that the fragmentation is a result of sheer failure in asymmetrical meteoroids. It seems no less probable that large meteoroids can contain planes of weakness and that the whole meteorite fails under compression at Öpik's destruction limit.

It should be noted, however, that the uniaxial crushing strength as determined in the laboratory is not a precise measure of the strength of the meteorite in flight. In the former case, pressure is applied on the ends of a cylinder until failure. However, if a lateral confining pressure is applied simultaneously to the sides of the cylinder, the compressive strength of the sample is increased, sometimes markedly. The meteorite in flight may be subjected to a stress field that is intermediate to those of the two laboratory conditions. For example, a spherical object will be subjected to a body force due to the drag but will also have an aerodynamic pressure normal to the surface and primarily over the leading hemisphere. We cannot yet estimate how much this particular confining pressure would increase the strength, if at all, but we suspect that Öpik's value, if it is a reasonable estimate of the effective crushing strength, is an upper limit to the classical uniaxial compressive strength. In Table I we give for the various parameters of equation (61) the maximum mass and the approximate height of maximum pressure for nonablating bodies ($\sigma = 0$) that can enter the atmosphere without structural failure.

We have used $\Gamma \rho V^2 = 2 \times 10^8$ dyn/cm^2 and an inverse atmospheric scale height of $b = 1.6 \times 10^{-6}$/cm. The latter value is an upper limit for the atmosphere where the major deceleration takes place and thus gives a lower limit to the masses quoted. For strong meteorite structures ($S_c = 3 \times 10^9$ dyn/cm^2) there is no limit in the mass that can reach sea level before crushing for any of the velocities considered.

Trajectories with low values of cos Z_R are statistically less likely as the velocity increases (Wood, 1961), and the observed limit of stony meteorite masses can be understood only if meteorites are relatively high-velocity objects for the most part. Wood reached the same conclusion with

Table I

Initial masses (m_∞) and masses (m) of meteorites of various velocities that attain a maximum load of 2×10^8 dyn/cm^2 at altitude H. Ablation is assumed to follow a law expressed by equation (12). Tabulated pressures are for vertical entry ($\cos Z_R = 1$)

Velocity V_∞ (km/sec)	Ablation coefficient σ (sec^2/cm^2)	Mass at maximum pressure $m(\cos Z_R)$ (g)	Height at maximum pressure H (km)	Mass at entry $m_\infty(\cos Z_R)$ (g)
	0	2.8×10^4	4.4	2.8×10^4
11	10^{-12}	2.39×10^4	4.7	3.5×10^4
	5×10^{-12}	1.75×10^4	5.8	9.0×10^4
	0	2.9×10^3	9.1	2.9×10^3
16	10^{-12}	2.06×10^3	9.8	4.4×10^3
	5×10^{-12}	6.95×10^2	12.2	9.5×10^3
	0	4.3×10^2	13.2	4.3×10^2
22	10^{-12}	2.48×10^2	14.3	9.6×10^2
	5×10^{-12}	8.6×10	16.7	7.3×10^3

totally different arguments based on the distribution of the local time of fall of meteorites.

There remain two other possible explanations for the absence of large meteoritic stones (we exclude the possibility that they do not exist in nature). First, the strengths are substantially less than the value we have assumed. Second, ablation may reduce the initial mass and, at the same time, increase the dynamic pressure available for crushing. The latter effect can be easily understood qualitatively if we note that a body that has ablated down to the critical mass will have a greater velocity at any given height than will a nonablating body of the same mass. The problem can be treated quantitatively for the special case of a constant ablation coefficient σ and a constant inverse scale height b. Equations (1) and (2) can be integrated to give the atmospheric density ρ_1 as a function of velocity V_1 according to

$$\Gamma\rho_1 = \frac{b \cos Z_R \, \rho_m^{2/3} \, e^{-u_0}}{2A} \int_{u_1}^{u_0} \frac{e^u}{u} \, du \quad , \tag{62}$$

where $u = \sigma V^2/6$. The pressure is given by

$$P_1 = \Gamma \rho_1 V_1^2 = \frac{3 \, b \cos Z_R \, \rho_m^{2/3} \, u_1 \, e^{-u_0} \, m_0^{1/3}}{2A\sigma} \int_{u_1}^{u_0} \frac{e^u}{u} \, du \tag{63}$$

Also, ·

$$\frac{dP}{du_1} = 0$$

when

$$e^{u_1} = \int_{u_1}^{u_0} \frac{e^u}{u} \, du = E_i(u_0) - E_i(u_1) \tag{64}$$

Using equation (12) and setting the maximum pressure $\rho_1 = 2 \times 10^8$ dyn/cm^2, we obtain

$$(65) \qquad m_1^{1/3} = \frac{10^8 \, \sigma A}{3 \, b \cos Z_R \, \rho_m^{2/3} \, u_1}$$

In Table I we show the limiting mass for survival for two values of the ablation coefficient; $\sigma = 10^{-12}$ sec^2/cm^2 is a lower limit for all photographic meteors, and $\sigma = 5 \times 10^{-12}$ sec^2/cm^2 is a value between an average meteor and the largest observed values (Jacchia, 1958). The preatmospheric masses m_∞ are computed from equation (12). A comparison of all results in Table I shows that ablation is not particularly effective in reducing the limiting size of the low-velocity objects and, therefore, is not an adequate explanation of the observed mass limit of stones.

It then follows from the preceding arguments that either (1) the effective crushing strength is $S_c \lesssim 10^7$ dyn/cm^2, or (2) meteorites do not arrive at the earth with $V \lesssim 16$ km/sec. Both possibilities contain interesting ramifications. Given a low crushing strength, we can suppose that the tensile strength is also low and we should reevaluate the limit for thermal shock derived in Section 3. If we presume that the decreased crushing strength is primarily due to planes of weakness in the material, rather than to a bulk property of the whole meteorite, then the exact mechanism of thermal fracture becomes crucial. The planes of weakness will act as stress relievers if our explanation for the existence of meteoritic stone less than the critical size for thermal fracture is correct, and the tensile strength for thermal stresses will be little affected. But should the preexisting cracks act as stress concentrators, the tensile strength would decrease greatly. We are disinclined to proceed further with this uncertain discussion and will only note that a decrease in the tensile strength of a factor of 5 will increase the limiting radius to 50 cm (equation (30)) or $m \approx 2 \times 10^3$ kg. Meteorites reach peak heating loads before peak dynamic loads, and thermal fracture in these large, weak structures can occur before compressive failure. To argue against a universal very low strength, we recall Buddhue's (1942) significant inverse relationship between the strength of meteorites and the number of stones associated with the fall. The strongest objects were almost always single falls. This fact precludes the possibility that the strength of the whole preatmospheric meteorite can be substantially less than that of the tested sample.

Let us now accept a lower limit of $V = 16$ km/sec in the explanation for the absence of large stones. We can establish a rough upper limit on the heat-transfer coefficient Λ for meteorites by taking the ablation fraction for the Saint-Severin meteorite proposed by Cantelaube, Pellas, Nordemann, and Tobailem (1969) on the basis of cosmic-ray tracks near the outer surface. Equation (12) gives

$$\Lambda = \frac{4\,\Gamma\,\zeta\,\ell n\,(m_0/m_e)}{V_0^2 - V_e^2} \quad , \tag{66}$$

where the subscript e refers to some final condition when ablation noticeably stops. We choose $V_e = 8$ km/sec as a very safe upper limit on the velocity (we have observed ablation at $V < 5$ km/sec in fireballs), and $\zeta = 8 \times 10^{10}$ ergs/g, the energy of vaporization. Cantelaube et al. give $m_0/m_e = 1.33$, and we find $\Lambda \leq 0.02$. If this low value of the heat-transfer coefficient is correct for meteorites, the near-negligible terminal mass of the fireballs we have observed offers an independent argument against their also possessing the physical characteristics of meteorites. A more realistic study of the ablation problem of a Saint-Severin-type body in which convective and radiative heat transfers are included would be an extremely useful addition to meteoritics. In particular, we would like to know the upper velocity limit consistent with the ablation defined by the cosmic-ray tracks.

In view of the complex nature of the attrition processes outlined above, we do not believe that any statement concerning the distribution of initial mass can be made by use of the observed distribution of meteorite masses. There is certainly no reason to believe that the final distribution should be a straight line in a log-mass — log-number plot.

Furthermore, if the initial mass distribution were known, our knowledge of the important details of the fragmentation process is too limited to predict any reliable distribution of meteorites on the ground. The mechanism of fracture or stress relief in thermal shock is ambiguous. The treatment of the ablation process and the heat-transfer problems has (in this paper and, generally, in the past) been sketchy at best. New methods developed for reentry problems could now be applied. In the case of pressure fragmentation, the concept of the usual uniaxial compressive strength is only a first-order approximation to the problem of an entering meteorite.

Finally, we note that the forces on a large meteoroid during entry are not trivial. For some material they are comparable to or greater than any forces the body has previously endured. The pressure load on large meteoroids is properly considered as a disruptive force. However, because of the hydrostatic normal force applied to the body behind the shock wave, there is also a compressive component that, for initially weak and low-density material, may cause sufficient compaction of some interior material to increase its strength. Can such indurate material arrive intact at the surface of the earth? Is the structure of a Type I carbonaceous chondrite consistent with its formation from a low-density (cometary?) body that has been compressed during atmospheric entry?

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LIST OF SYMBOLS

$\mathcal{A}$ — frontal area of the meteoroid.

A — shape factor.

b — inverse density scale height in the atmosphere.

c — heat capacity.

D — distance along the meteor trajectory.

f — parameter describing the direction of departing ablation products.

F, G — functions defined by equations (25) and (26).

I, i — luminous intensity within a specified spectral region.
Note: When only undispersed photographic information is available, intensities are conventionally expressed in terms of the brightness of an A0 star producing the equivalent photographic effect. These intensities, given relative to a 0-magnitude star, are defined as $I = -2.5 \log M$.

K — $= 2^{-1/3} \Gamma A \tau_0^{1/3} \rho_m^{-2/3}$.

m — instantaneous mass of the meteoroid.

m_∞ — initial mass.

m_d — dynamic mass, determined from the measured drag.

m_p — photometric mass, determined from the measured luminosity.

m_t — terminal mass.

m_v — mass lost by vaporization.

r — radius vector of a spherical meteoroid.

R — radius of a spherical meteoroid.

R_{lim} — radius of a meteoroid for which the compressive stress at the surface and the tensile stress at the center are both equal to the corresponding strength of the meteoroid material.

S_t, S_c	tensile, compressive strength of the meteoroid.
T, T_0	absolute temperature inside and outside the atmosphere.
V	instantaneous meteoroid velocity.
V_∞	initial meteoroid velocity.
V_c	meteoroid velocity with which the critical temperature τ_c is reached.
w	speed of ablation products with respect to the meteoroid.
W	W^{-1} is a characteristic depth of heating (equation (21)).
y	Young's modulus.
Z_R	angle between the meteor trajectory and the local vertical.
α	thermal expansion coefficient.
β	$= (\lambda/\rho_m c)^{1/2}$.
Γ	drag coefficient. Note: We use $\Gamma = 1$ (free molecular flow, faint meteors) and $\Gamma = 0.46$ (continuum flow, fireballs).
ϵ	thermal emissivity.
Λ	heat-transfer coefficient.
λ	heat conductivity.
ν	Poisson's ratio.
Π	thermal stress in the meteoroid in a direction perpendicular to **r**.
ρ	atmospheric density.
ρ_m	meteoroid density.
$\underline{P}$	thermal stress in the meteoroid along the radius.
ρ_c	air density at which the critical temperature τ_c is reached.
σ	$= \Lambda/2\Gamma\rho$, ablation coefficient.
σ_R	Stefan-Boltzmann constant.
τ	temperature relative to T_0.
τ_c	critical surface temperature at which the meteoroid material strength is reached.

τ_0	luminosity coefficient.
χ	fragmentation index (equation (53)).
ξ	heat of ablation.

BIOGRAPHICAL NOTES

RICHARD E. McCROSKY received his B.S. degree in physics from Harvard University in 1952 and his Ph.D. in astronomy from that university in 1956.

Dr. McCrosky holds joint appointments as Astronomer, Smithsonian Astrophysical Observatory, and Research Associate, Harvard University. He is also Scientist-in-Charge of the Smithsonian's optical meteor projects. His primary research specialties include photographic and spectral meteor studies.

ZDENĚK CEPLECHA received the RNDr. degree from Charles University, Czechoslovakia, in 1952 and the C.Sc. and D.Sc. degrees from the Czechoslovak Academy of Sciences in 1955 and 1967, respectively.

Since 1951 Dr. Ceplecha has been an astrophysicist with the Astronomical Institute of the Czechoslovak Academy of Sciences in Ondřejov. In 1968 he held a National Research Council Postdoctoral Visiting Research Associateship at the Smithsonian Astrophysical Observatory.

Dr. Ceplecha's principal field of investigation is meteors and meteoroids.

NOTICE

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